Randomized Algorithms

- Deterministic and (Las Vegas & Monte Carlo) Randomized Algorithms
- Probability Review
- Probabilistic Analysis of deterministic QUICK-SORT Algorithm
- RANDOMIZED-SELECT and RANDOMIZED-QUICK-SORT
- Max-Cut
- Min-Cut
- MAX-3-SAT and Derandomization
- Closest pair
- Hashing, Bloom filters, Streams, Sampling, Reservoir sampling, Sketch

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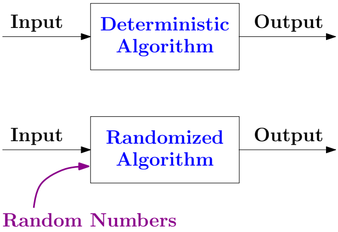
Algorithm Design Paradigms

- Greedy
- Divide-and-Conquer
- Dynamic Programming
- Network Flows

We have only seen **Deterministic Algorithms** so far

Randomized Algorithms incorporate randomness in their operation



Randomized Algorithm receives, in addition to the input, a random number stream to make random decisions during execution

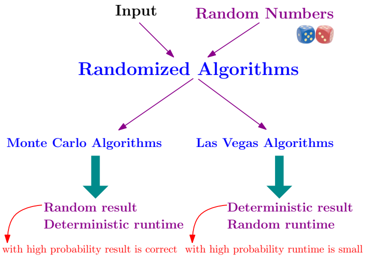
▷ May give different results on the same input in different runs

Often aims for properties like:

- Good average-case (expected) behavior
- Getting exact answers with high probability
- Getting answer close to the right answer with high probability
- Runtime is small with high probability

Advantages:

- Simple and elegant. Their output/runtime is good with high probability
- The execution time or space requirement is smaller than that of the best deterministic algorithm



- **Monte Carlo Algorithms**
  - Guaranteed to run in a fixed time
  - Output is random
    - ▷ Output is correct with high probability
  - e.g. Min cut algorithm
- **Las Vegas Algorithms**
  - Guaranteed to output the correct answer
  - Running time is random
    - ▷ Runtime is small with high probability
  - e.g: randomized quicksort, Closest pair

Input: An array A with $n/4$ 1's and $3n/4$ 0's

Output: An index k such that $A[k] = 1$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
0	0	0	1	0	0	0	0	0	0	1	0	0	1	1	0

Algorithm Deterministic

```
k ← 0
for i = 1 → n do
    if A[i] = 1 then
        k ← i
        return k
```

Algorithm Monte Carlo

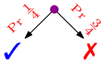
```
k ← RANDOM(1 ··· n)
return k
```

Algorithm Las Vegas

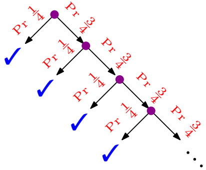
```
k ← 1
while A[k] ≠ 1 do
    k ← RANDOM(1 ··· n)
return k
```

Quality: correct
worst case runtime: $\frac{3n}{4}$

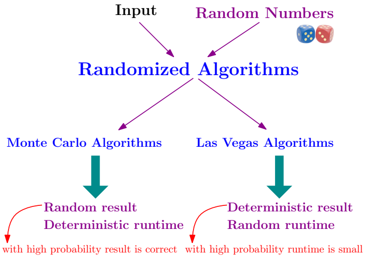
Quality: correct w.p $\frac{1}{4}$
worst case runtime: 1



Quality: correct
expected runtime: 4



Las Vegas and Monte Carlo Algorithms



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Can always convert a Las-Vegas algorithm into a Monte Carlo algorithm

- Stop the algorithm after a certain point
- but no method is known for the other way - needs efficient verification