Intractable Problems

- Clique
- Independent Set
- Vertex Cover
- Set Cover
- Set Packing
- Satisfiability Problem
- Hamiltonian Cycle and Path
- Traveling Salesman Problem
- Graph Coloring
- Circuit Satisfiability
- Knapsack
- Subset Sum
- Prime and Factor
- Partition

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KNAPSCAK and SUBSET-SUM Problem

- Given a set $U = \{a_1, a_2, \ldots, a_n\}$ of objects
- A weight function $w : U \rightarrow \mathbb{Z}^+$ $\triangleright\ w_i = w(a_i)$
- A value function $v : U \rightarrow \mathbb{R}^+$ $\triangleright\ v_i = v(a_i)$
- And a positive integer C

KNAPSACK(U, w, v, C, k) problem:

Is there a $S \subset U$ such that $\sum\limits_{a_i \in S} w_i \leq C$ and $\sum\limits_{a_i \in S} v_i = k$?

SUBSET-SUM(U, w, C) problem: Is there a $S \subset U$ such that $\sum\limits_{a_i \in S} w_i = C$?

Number Theory Problems

PRIME(n) problem: Is the integer n a prime?

COMPOSITE(n) problem: Is the integer n a composite number?

FACTOR(n, k) problem: Is there a factor d of n such that $2 \leq d \leq k$?

These problems are the building blocks of public key cryptography

The PARTITION Problem

- Given a set $U = \{a_1, a_2, \ldots, a_n\}$ of n positive integers
- Partition U into two subsets U_1 and U_2
- $\sum\limits_{a \in U_1} a = \sum\limits_{a \in U_2} a$ $\triangleright$ balanced partition
- Also called the number bipartition problem
- If $\left| \sum\limits_{a \in U_1} a - \sum\limits_{a \in U_2} a \right| = k$, then it is called k-imbalanced bipartition

PARTITION(U, k) problem: Is there a k-imbalanced biparition of U?

Childhood team selection actually was a greedy algorithm for bipartition

Multiway partition problem is an interesting extension of this problem

PARTITION Problem: Applications

Multiprocessor Scheduling

Assign tasks to two identical processors to minimize the MAKESPAN

MAKESPAN is the latest finishing processor

- U is the set of tasks
- A balanced bipartition of U (tasks in each part to be run on one processor) minimizes the makespan

For k processors, it becomes the k-way partition problem

PARTITION Problem: Applications

Scoring Based Voting

Three candidates A, B, C and voters with weighted votes. Each voter votes to veto a candidate. The candidate with the smallest total weight of vetoes wins.

We have a subset of n voters with weights $a_1, a_2, \ldots, a_n$, who wants to select candidate A. How should they cast their vetoes to ensure A wins.

- $U = \{a_1, a_2, \ldots, a_n\}$
- U wants to elect A, they should "bipartition" their vetoes for B and C
- This will maximize the minimum vetoes for B and C