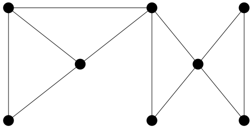
Intractable Problems

- Clique
- Independent Set
- Vertex Cover
- Set Cover
- Set Packing
- Satisfiability Problem
- Hamiltonian Cycle and Path
- Traveling Salesman Problem
- Graph Coloring
- Circuit Satisfiability
- Knapsack
- Subset Sum
- Prime and Factor
- Partition

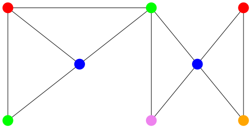
Imdad ullah Khan

Graph Coloring

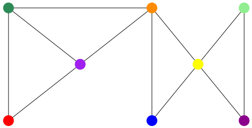
A graph (vertex) coloring is to assign a color to each vertex such that no two adjacent vertices get the same color



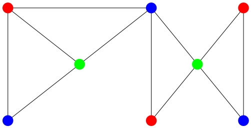
A graph G on 8 vertices



A coloring with 6 colors



A coloring with 8 colors



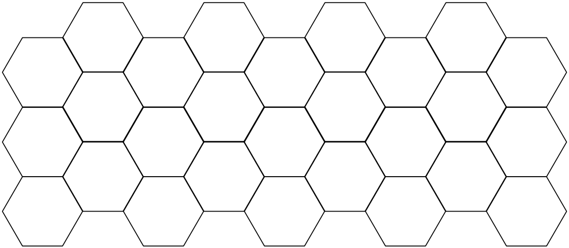
A coloring with (optimal) 3 colors

k-COLORING(G) problem: Is there a coloring of G with k colors?

Graph Coloring Applications

GSM Frequency Bands Assignment

- In cellular networks (GSM) coverage area is divided into a hexagonal grid
- Each cell (a hexagon) is served by an antenna
- Each cell uses a frequency band (one of 850, 900, 1800, 1900 MHz)
- Frequency of a cell must be different from adjacent cells (hexagons sharing a line segment)
- 4-color vertices of the dual graph of the hexagonal grid



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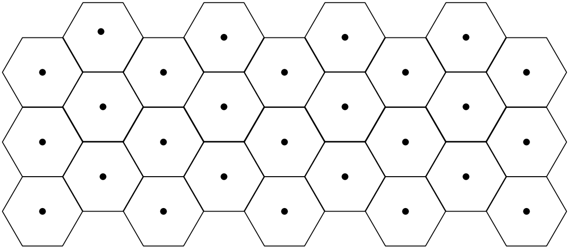

Graph Coloring Applications

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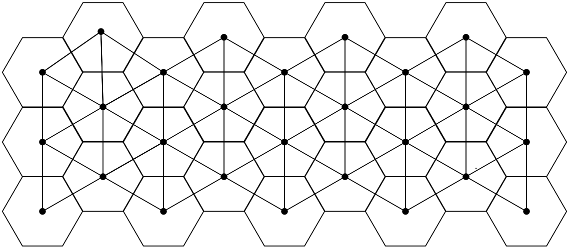

Graph Coloring Applications

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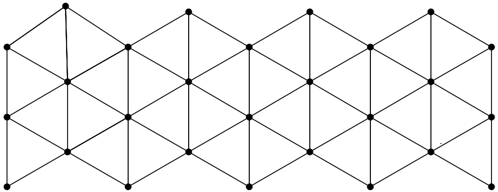

Graph Coloring Applications

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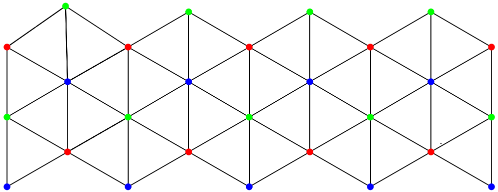

Graph Coloring Applications

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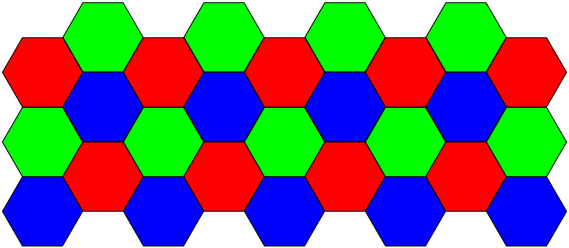
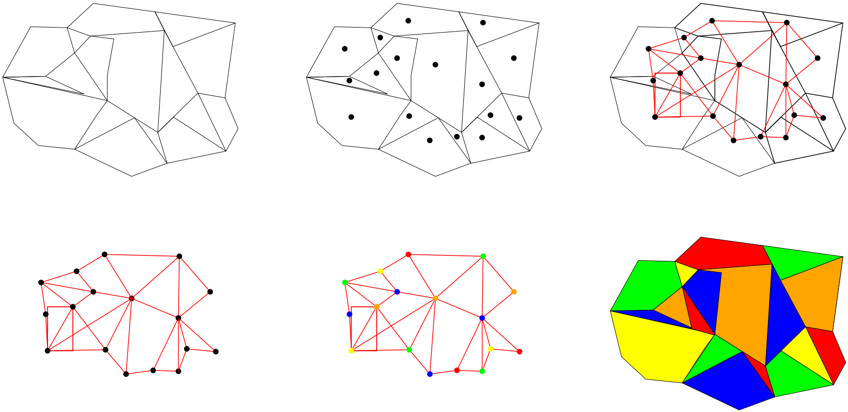


Graph Coloring Applications

Map Coloring

- Color regions of map
- No neighboring regions can have the same color

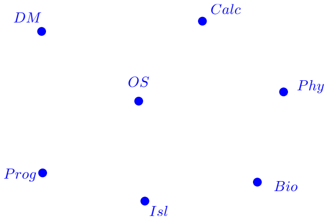


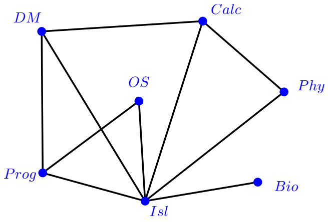
Graph Coloring Applications

Final Exam Scheduling

Optimally schedule *n* exam with no student having > 1 parallel exam

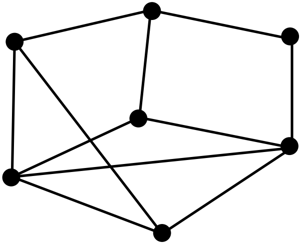
- Make graph on courses with common students encoded as edges
- Find the minimum number of colors needed to color the graph

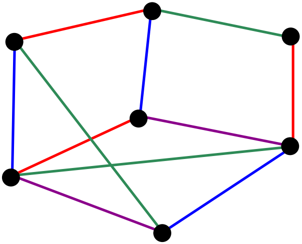




Edge Coloring

An edge coloring of a graph is to assign a color to each edge such that no two "adjacent edges" get the same color





k-EDGE-COLORING(G) problem: Is there edge-coloring of G with k colors?

Edge Coloring Applications

NFL season scheduling

n teams playing in a tournament. Based on last year's record, each team will play some other teams. We want to determine a schedule with as few rounds as possible

- Make a node for each team
- An edge for each game to be played
- Find an edge coloring with minimum number of color

Edge Coloring Applications

Open Shop Scheduling (time division multi-processing)

n objects to be manufactured. Manufacturing object o_i entails performing tasks $t_{i1}, \ldots, t_{ij_i}$ (in any order). Each task requires one of non-parallel machines $M_1, \ldots, M_k$. We want to schedule machines usage to manufacture all n objects in least time.

- Make a (multi) bipartite graph [Objects, Machines] edges
- An edge o_i, m_j edge means object i has a task requiring machine m_j
- An edge coloring with minimum number of colors (time slots)