# Intractable Problems

- Clique
- Independent Set
- Vertex Cover
- Set Cover
- Set Packing
- Satisfiability Problem
- Hamiltonian Cycle and Path

- Traveling Salesman Problem
- Graph Coloring
- Circuit Satisfiability
- Knapsack
- Subset Sum
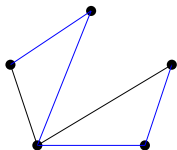- Prime and Factor
- Partition
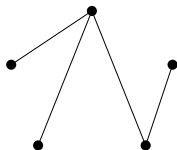
Imdad ullah Khan

(Directed) Hamiltonian Cycle and Path

A Hamiltonian cycle (path) in graph is a cycle (path) containing all vertices
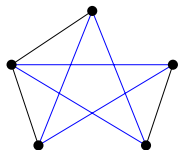
Hamiltonian cycle in G

No Hamiltonian cycle in G
Hamiltonian path in blue

No Hamiltonian path in G
So no hamiltonian cycle

HAM-CYCLE(G) problem: Does G have a Hamiltonian cycle?

HAM-PATH(G) problem: Does G have a Hamiltonian path?

DIR-HAM-CYCLE(G), and DIR-HAM-PATH(G) are defined analogously

Hamiltonian Cycle and Path Applications

Is there a sequence of moves that takes the knight to each square on an 8×8 chessboard exactly once, returning to the original square?

- For 8×8 Abu Bakr Muhammad b. Yahya al-Suli found one in 9th century

- For $n \times n$ chessboard define a vertex for each position and connect vertex v_{ij} to vertex v_{kl} if there is a legal move between the (i,j)th position to the (k,l)th position on the board

- Find a Hamiltonian cycle in the graph
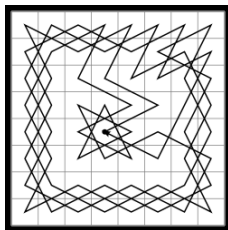
Hamiltonian Cycle and Path Applications

Route for School Bus

School bus should visit each house exactly once to save fuel and time

- Houses considered nodes and streets as edges
- Find a Hamiltonian cycle

Hamiltonian Cycle and Path Applications

Genome Mapping

Combine many tiny fragments of genetic codes (called "reads"), into one genomic sequence

- Consider each read a node in a graph
- Overlap (end of one read matches the start of another) is an edge
- Find a Hamiltonian cycle in this graph, a mapping of genome
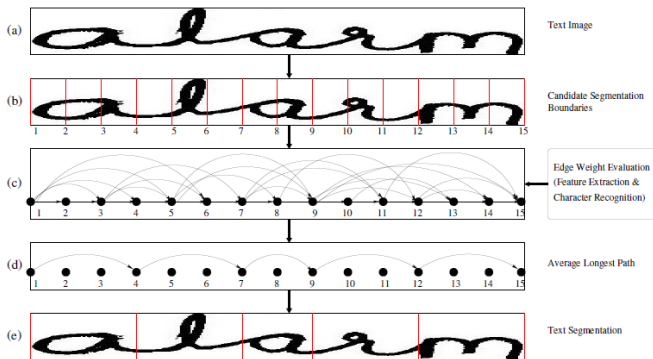
Longest Path Problem

- Given an edge weighted graph $G = (V, E, w)$, two vertices s and t

- The longest $s - t$ path P is a path from s to t with maximum total weight

LONGEST-PATH(G, s, t, k) problem: Is there a s-t path of weight $\geq k$?

Longest Path Problem: Application

Character Segmentation for Optical Character Recognition:

- First step in any OCR system is character segmentation
- Isolate individual characters in hand-written text
- Input to character recognition system
- Salvi et.al. (2013) proposed algorithm based on average longest paths

Longest Path Problem: Application

Static timing analysis (STA)

A simulation method of computing the expected timing of a digital circuit without requiring to simulate the full circuit. STA is performed only on these critical paths

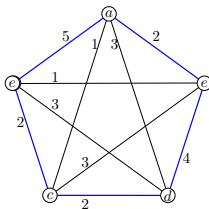> ▷ Widely used method in circuit design and embedded systems

- Consider circuit components as graph nodes
- Connections between components are edges
- Find longest path between input and output component
- Longest path identifies critical paths in an IC or VLSI system
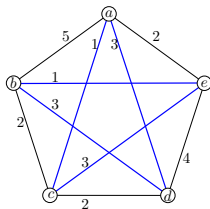
Traveling Salesman Problem

Given a complete graph G on n vertices with edge weights, a TSP tour is a Hamiltonian cycle in G
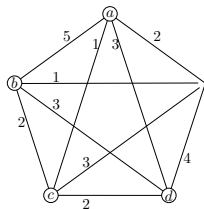
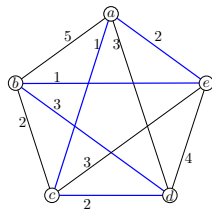K_5 with edge weights   A TSP tour of length 15   A TSP tour of length 11   A TSP tour of length 9

Traveling Salesman Problem TSP(G, w): Is there a TSP tour of weight w?

TSP Applications

- **Transportation:** A salesman wants to visit all cities with minimum cost

- **Optimize the tool path** for manufacturing equipment

DRILL HOLES