

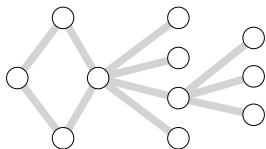
Intractable Problems

- Clique
- Independent Set
- Vertex Cover
- Set Cover
- Set Packing
- Satisfiability Problem
- Hamiltonian Cycle and Path
- Traveling Salesman Problem
- Graph Coloring
- Circuit Satisfiability
- Knapsack
- Subset Sum
- Prime and Factor
- Partition

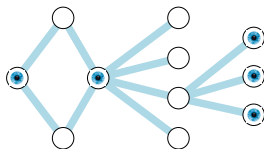
IMDAD ULLAH KHAN

Vertex Cover

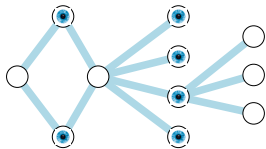
An **vertex cover** in a graph is subset C of vertices such that each edge has at least one endpoint in C



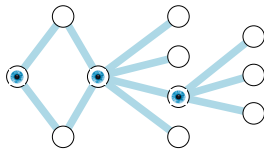
A graph on 11 vertices



A vertex cover of size 5



A vertex cover of size 6



A vertex cover of size 3

The **VERTEX-COVER**(G, k) problem: **Is there a cover of size k in G ?**

Vertex Cover Application

Network Security: Rout Based Filtering

- Identify a small set of routers/AS
- So as all packets can be monitored at those routers/switches
 - ▷ check if the source/destination addresses are valid given the routing table and network topology
- Route-based distributed packet filtering
- Prevents distributed denial of service attacks
- Represent the network topology as a graph
- Find the smallest vertex cover

Set Cover

- Given a set U of n elements
- A collection \mathcal{S} of m subsets $S_1, S_2, \dots, S_m \subseteq U$
- A **Set Cover** is a subcollection $I \subset \{1, 2, \dots, m\}$ with $\bigcup_{i \in I} S_i = U$

$$U := \{1, 2, 3, 4, 5, 6\}$$

$$\mathcal{S} := \{\{1, 2, 3\}, \{3, 4\}, \{1, 3, 4, 5\}, \{2, 4, 6\}, \{1, 3, 5, 6\}, \{1, 2, 4, 5, 6\}\}$$

$$\text{Cover-1: } \{1, 2, 3\}, \quad \{1, 3, 4, 5\}, \{2, 4, 6\}$$

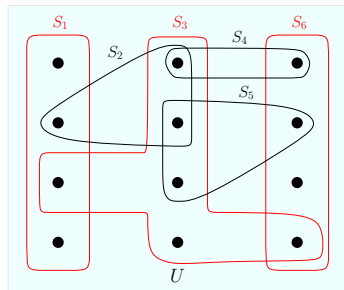
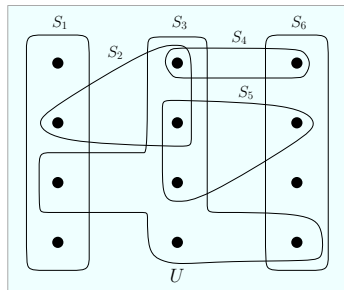
$$\text{Cover-2: } \{1, 2, 3\}, \quad \{1, 3, 4, 5\}, \{2, 4, 6\}, \{1, 2, 4, 5, 6\}$$

$$\text{Cover-3: } \{1, 3, 4, 5\}, \quad \{1, 2, 4, 5, 6\}$$

Cover-1 has size 3, the latter two have size 2 each

Set Cover

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The **SET-COVER**(U, \mathcal{S}, k) problem: Is there a cover of size k for U ?

Set cover applications

System Integration

Build a software systems meeting the requirement specification from available software tools

- U is the set of capabilities we want our system to have
- \mathcal{S} : Available softwares each providing a **subset** of capabilities
- Select a (small) subset of softwares to provide all functionalities

Set cover applications

IBM antivirus tool

Detect virus in codes

- They found a set of about 9000 strings of 20 bytes or more that occur in the binaries of viruses but not in “clean” codes
- U is the set of (500) known viruses
- \mathcal{S} : For each string the subset of viruses containing it
- Select a (small) subset of strings that should be searched in codes to detect any virus

Set Packing

- Given a set U of n elements
- A collection \mathcal{S} of m subsets $S_1, S_2, \dots, S_m \subseteq U$
- A subcollection $I \subset \{1, 2, \dots, m\}$ pack together if for all $i \neq j \in I$
 $S_i \cap S_j = \emptyset$

$U := \{1, 2, 3, 4, 5, 6\}$

$\mathcal{S} := \{\{1, 2, 3\}, \{4, 5\}, \{4, 6\}, \{2, 3\}, \{1, 6\}, \{4, 5, 6\}\}$

Pack-1: $\{1, 2, 3\}, \{4, 5\}$

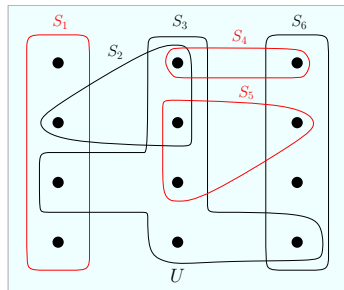
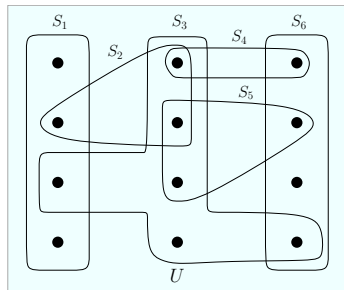
Pack-2: $\{4, 5\}, \{2, 3\}, \{1, 6\}$

Pack-3: $\{1, 2, 3\}, \{4, 5, 6\}$

Pack-1 and Pack-3 have size 2 each, Pack-2 has size 3

Set Packing

- Given a set U of n elements
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- A subcollection $I \subset \{1, 2, \dots, m\}$ pack together if for all $i \neq j \in I$ $S_i \cap S_j = \emptyset$



The **SET-PACKING**(U, \mathcal{S}, k) problem: Is there a packing of size k ?

Set packing Applications

Resource Sharing

Process scheduling using limited resources

- U is a set of non shareable resources
- \mathcal{S} : Processes each requesting a subset of resources
- Select a (large) subset of processes to allocate resources to

Set packing Applications

Airline Crew Scheduling

Make a roster of staff to service a set of flights

- U is the set of crew staff
- \mathcal{S} : teams of members willing to work with each other
- Select the most number of teams each serving a flight