Dynamic Programming

- (Weighted) Independent Set in Graphs
- Weighted Independent Sets in Path
- Dynamic Programming Formulation
- Implementation and Backtracking

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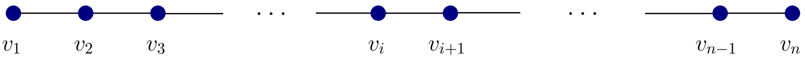
The Path Graph

The path graph is a connected graph with two nodes of degree 1 and the other $n-2$ vertices of degree 2



$$\text{Number of edges} = \frac{1 + 2(n-2) + 1}{2} = n-1$$

▷ So a path is a tree

Max weight independent set in path graph

Input: A node weighted graph $G = (V, E), w$ $w : V \to \mathbb{R}^+$

Output: An independent set of G of maximum ~~cardinality~~ weight

A company wants to open restaurants on the motorway

- Designated service areas $s_1, \ldots, s_n$ every 7 kilometers
- A restaurant at s_i gives estimated profit p_i
- No two restaurants can be located within 10 km of each other

Select a subset of sites to maximize total profit

Problem can be modeled by a node weighted path graph

- Each site s_i is a vertex with weight equal to p_i
- If two sites are within 10 km of each other make an edge between the corresponding vertices ▷ note: we get a path graph

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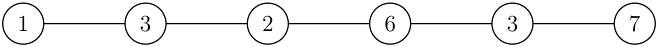
Output: An independent set of P of maximum weight

No consecutive vertices can be chosen



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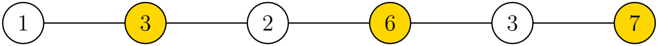

An independent set of weight 16

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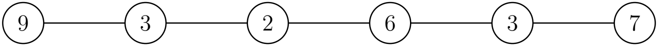

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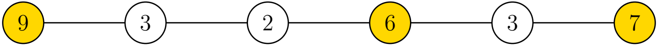

An independent set of weight 22

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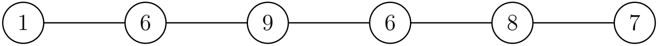
Greedy Approach:

- Select a node with max weight
- Mark its neighbors as incompatible
- Repeat the process with remaining unmarked nodes



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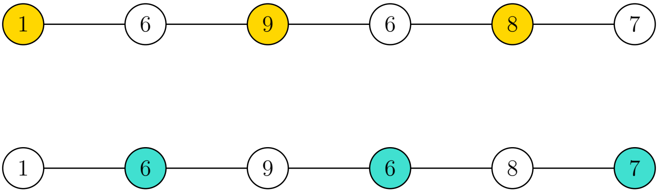

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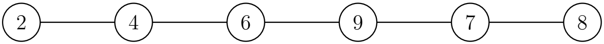
Divide & Conquer approach-1:

- Divide P into left and right halves
- Find max weight independent sets in both
- Combine the two sets to get the answer



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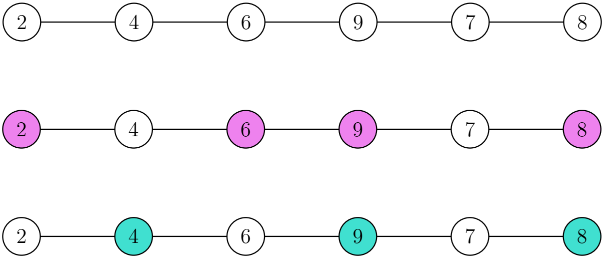

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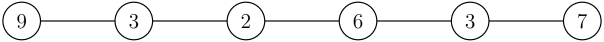
Divide & Conquer approach-2:

- Divide P into odd and even indexed vertices
- Each one is an independent set
- Return the larger of the two



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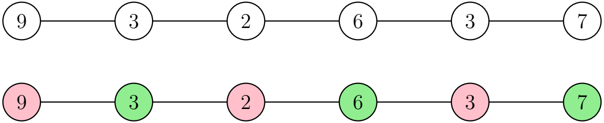

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