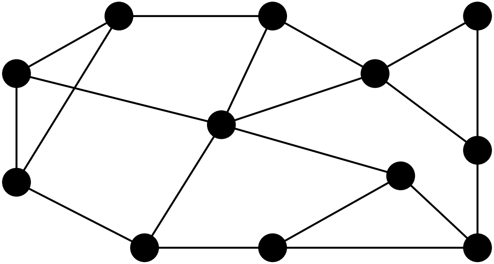
Dynamic Programming

- (Weighted) Independent Set in Graphs
- Weighted Independent Sets in Path
- Dynamic Programming Formulation
- Implementation and Backtracking

Imdad ullah Khan

Independent Set in Graph

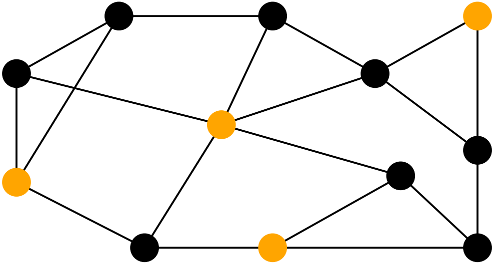
An independent set in a graph G is a subset of vertices no two of which are adjacent



Graph $G = (V, E)$

Independent Set in Graph

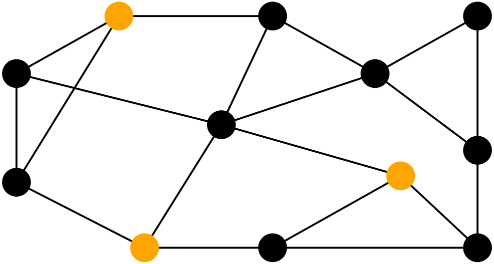
An independent set in a graph G is a subset of vertices no two of which are adjacent



An independent set of size 4

Independent Set in Graph

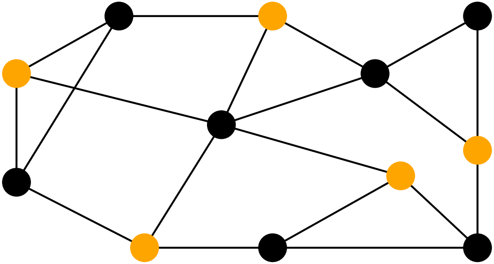
An independent set in a graph G is a subset of vertices no two of which are adjacent



An independent set of size 3

Independent Set in Graph

An independent set in a graph G is a subset of vertices no two of which are adjacent



Looking for a largest independent set

The Maximum Independent Set Problem

Input: A graph $G = (V, E)$

Output: An independent set of G of maximum cardinality

Applications in scheduling, resource allocation, VLSI design

This problem is very hard!

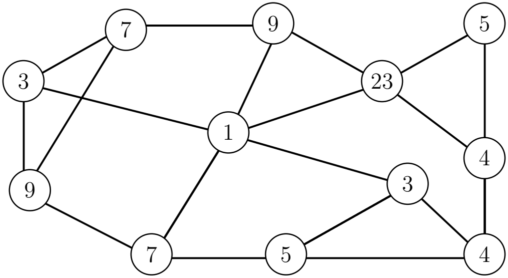
- No known polynomial time algorithm for it
- Essentially, the brute force algorithm is the best known
- We will show that this is a NP-HARD problem

Next we discuss an even harder version of it

Weighted Independent Set

Given a node-weighted graph $G = (V, E)$, $w : V \to \mathbb{R}$

Weight of $S \subset V$: sum of weights of vertices in S

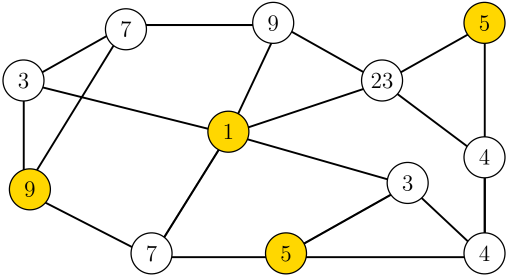


A node weighted graph

Weighted Independent Set

Given a node-weighted graph $G = (V, E)$, $w : V \to \mathbb{R}$

Weight of $S \subset V$: sum of weights of vertices in S

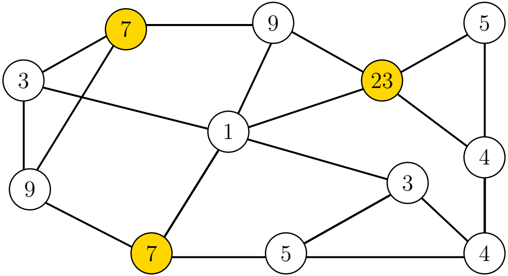


A maximal independent set with weight 20 $\triangleright$ cannot add to it

Weighted Independent Set

Given a node-weighted graph $G = (V, E)$, $w : V \to \mathbb{R}$

Weight of $S \subset V$: sum of weights of vertices in S

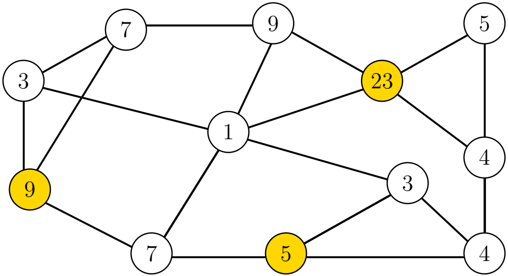


A non-maximal independent set with weight 37

Weighted Independent Set

Given a node-weighted graph $G = (V, E)$, $w : V \to \mathbb{R}$

Weight of $S \subset V$: sum of weights of vertices in S

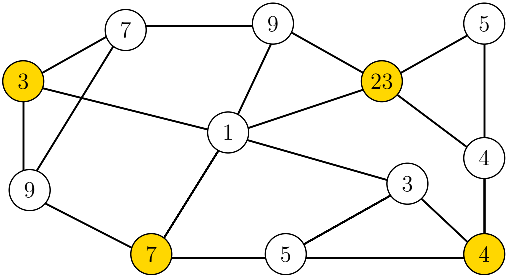


A maximal independent set with weight 37

Weighted Independent Set

Given a node-weighted graph $G = (V, E)$, $w : V \to \mathbb{R}$

Weight of $S \subset V$: sum of weights of vertices in S



A maximal independent set with weight 37

The Maximum Weight Independent Set Problem

Input: A node weighted graph $G = (V, E), w$ $w : V \to \mathbb{R}^+$

Output: An independent set of G of maximum ~~cardinality~~ weight

The problem is harder than maximum independent set problem!

- Max independent set is it's special case
  - ▷ Can use solution to max WIS to solve max independent set problem
- This is clearly NP-HARD problem