Polynomial Time Reduction

- Polynomial Time Reduction Definition
- Reduction by Equivalence
- Reduction from Special Cases to General Case
- Reduction by Encoding with Gadgets
- Transitivity of Reductions
- Decision, Search and Optimization Problem
- Self-Reducibility

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Versions of Problems: Decision Problems

Decision Problem

Sometimes called decision version of a problem

- These problems can be characterized by their algorithms whose output is either **Yes** or **No**
- In other words the answer on an instance is either **Yes** or **No**

SAT, 3-SAT are decision problems

So are all the other problems we studied so far

IND-SET(G, k), VERTEX-COVER(G, k), PRIME(n),
CLIQUE(G, k), SET-COVER$(U, \mathcal{S}, k)$, SUBSET-SUM(U, w, C)

Versions of Problems: Search Problems

Search Problem

Sometimes called search version of a problem

- These problems ask for a structure satisfying certain property or NOT-FOUND= **NF** flag
- The expected answer on an instance is not (necessarily) **Yes** or **No**

Search versions of SAT, 3-SAT ask for a satisfying assignment

- output is n-bit string (specifying ordered values for variables) or **NF**

Search version of IND-SET(G, k) asks for an ind. set of size k in G

- output is a subset of vertices or **NF**

Search version of SET-COVER$(U, \mathcal{S}, t)$

- output is a t-sized sub collection of $\mathcal{S}$ or **NF**

Versions of Problems: Optimization Problems

Optimization Problem

Sometimes called optimization version of a problem

- These problems ask for a structure that satisfy certain property (feasibility) and no other feasible structure have better value
- These are search problem but searching for an optimal structure

Optimization versions of SAT, 3-SAT ask for an assignment satisfying the most number of clauses

- output is n-bit string (specifying ordered values for variables)

MAX-IND-SET(G), MAX-CLIQUE(G) ask for largest independent set or clique in a graph G

MIN-VERTEX-COVER(G) asks for a vertex cover of minimum size

TSP(G) asks for a minimum cost TSP tour

As in DP, sometimes we only need value of the optimal solution

Versions of Problems

- Decision Problem: answer is **Yes/No**
- Search Problem: answer is a feasible structure of certain value or **NF**
- Optimization Problem: answer is a feasible structure of optimal value

Some authors only use decision problems and search problems.

▷ Search problem there actually means the optimization problem

This is perhaps a better notion, since if you know value of the optimal solution (which can be found through decision version of the problem), then one can use search problem (our notion) with the input value equal to the optimal value

In some cases there is no reasonable notion of optimization version e.g. Hamiltonian cycle problem and 3-coloring problem

Versions of Problems: Self Reducibility

Are versions of a problem polynomial time reducible to each other?

Many search and optimization problems are only polynomially more difficult than corresponding decision problem

▷ Any efficient algorithm for the decision problem can be used to solve the search problem efficiently

This is called **self-reducibility**

All the problems we discuss exhibit self-reducibility