Polynomial Time Reduction

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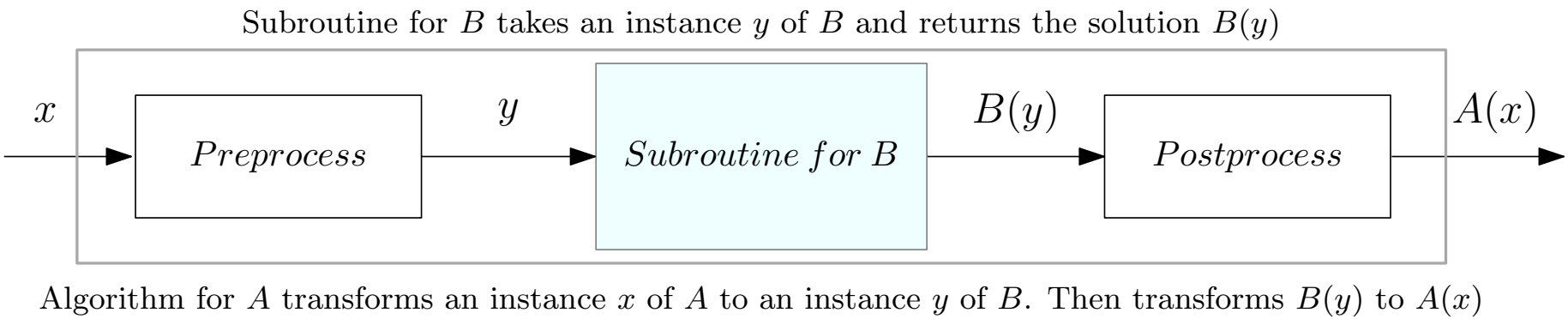
Polynomial Time Reduction

Problem A is polynomial time reducible to Problem B, $A \leq_p B$

If any instance of problem A can be solved using a polynomial amount of computation plus a polynomial number of calls to a solution of problem B

If any algorithm for problem B can be used [called (once or more) with *'clever'* legal inputs] to solve any instance of problem A



Algorithm for A transforms an instance x of A to an instance y of B. Then transforms $B(y)$ to $A(x)$

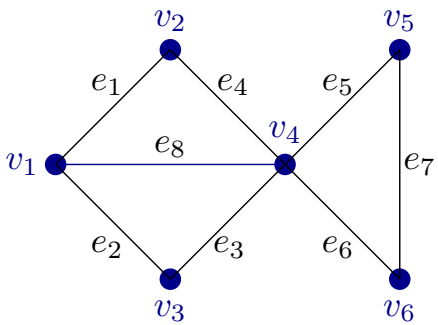
Reduction from special case to general case

$$\text{VERTEX-COVER}(G, k) \leq_p \text{SET-COVER}(U, \mathcal{S}, k')$$

Let $\mathcal{A}$ be an algorithm solving $\text{SET-COVER}(U, \mathcal{S}, k')$

Let $[G, k]$ be an instance of the VERTEX-COVER problem

1. Make $U = E, \qquad k' = k$
2. $\mathcal{S} = \{S_1, \ldots, S_n\}$, where $S_i = \{e \in E \mid e \text{ is incident on } v_i\}$



$$U = \{e_1, e_2, e_2, e_4, e_5, e_6, e_7\}$$

$$S_1 = \{e_1, e_2, e_8\}$$
$$S_3 = \{e_2, e_3\}$$
$$S_2 = \{e_1, e_4\}$$
$$S_4 = \{e_3, e_4, e_5, e_6, e_8\}$$
$$S_5 = \{e_5, e_7\}$$
$$S_6 = \{e_6, e_7\}$$

Theorem: $[U, \mathcal{S}]$ has a set cover of size k iff G has a vertex cover of size k

3. If $\mathcal{A}(U, \mathcal{S}, k') =$ **Yes**, then output **Yes**, else output **No**

Reduction from special Case to General Case

We get the following reduction very similarly

$$\text{INDEPENDENT-SET}(G, k) \leq_p \text{SET-PACKING}(U, \mathcal{S}, k')$$

The following reductions are even more straight forward. They follow from respective definitions of the problems

$$3\text{-SAT}(f) \leq_p \text{SAT}(f')$$

$$\text{SUBSET-SUM}(U, w, C) \leq_p \text{KNAPSACK}(U, w, v, C')$$

Please complete their details. Explicitly and formally writing them will help understand the important notion of reduction