

## Intractable Problems

- Clique
- Independent Set
- Vertex Cover
- Set Cover
- Set Packing
- Satisfiability Problem
- Hamiltonian Cycle and Path
- Traveling Salesman Problem
- Graph Coloring
- Circuit Satisfiability
- Knapsack
- Subset Sum
- Prime and Factor
- Partition

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# KNAPSCAK and SUBSET-SUM Problem

- Given a set  $U = \{a_1, a_2, \dots, a_n\}$  of objects
- A weight function  $w : U \rightarrow \mathbb{Z}^+$  ▷  $w_i = w(a_i)$
- A value function  $v : U \rightarrow \mathbb{R}^+$  ▷  $v_i = v(a_i)$
- And a positive integer  $C$

KNAPSACK( $U, w, v, C, k$ ) problem:

Is there a  $S \subset U$  such that  $\sum_{a_i \in S} w_i \leq C$  and  $\sum_{a_i \in S} v_i = k$ ?

SUBSET-SUM( $U, w, C$ ) problem: Is there a  $S \subset U$  such that  $\sum_{a_i \in S} w_i = C$ ?

# Number Theory Problem

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PRIME( $n$ ) problem: Is the integer  $n$  a prime?

COMPOSITE( $n$ ) problem: Is the integer  $n$  a composite number?

FACTOR( $n, k$ ) problem: Is there a factor  $d$  of  $n$  such that  $2 \leq d \leq k$ ?

These problems are the building blocks of public key cryptography

# The PARTITION Problem

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- Given a set  $U = \{a_1, a_2, \dots, a_n\}$  of  $n$  positive integers
- Partition  $U$  into two subsets  $U_1$  and  $U_2$
- $\sum_{a \in U_1} a = \sum_{a \in U_2} a$  ▷ balanced partition
- Also called the number bipartition problem
- If  $\left| \sum_{a \in U_1} a - \sum_{a \in U_2} a \right| = k$ , then it is called  $k$ -imbalanced bipartition

**PARTITION( $U, k$ )** problem: Is there a  $k$ -imbalanced bipartition of  $U$ ?

Childhood team selection actually was a greedy algorithm for bipartition

Multiway partition problem is an interesting extension of this problem

### Multiprocessor Scheduling

- Assign tasks to two identical processors
- The goal is to minimize the MAKESPAN
- MAKESPAN is the latest finishing processor
- Could have many processors, need to use  $k$ -way partition

### Scoring Based Voting

- Three candidates  $A, B, C$
- $U$ :  $n$  voters (among others) with weighted votes  $a_1, a_2, \dots, a_n$
- Each voter votes to veto a candidate
- Candidate with the smallest total weight of vetoes wins
- $U$  wants to elect  $A$ , they should “bipartition” their votes for  $B$  and  $C$
- This will maximize the minimum vetoes for  $B$  and  $C$