

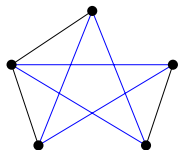
Intractable Problems

- Clique
- Independent Set
- Vertex Cover
- Set Cover
- Set Packing
- Satisfiability Problem
- Hamiltonian Cycle and Path
- Traveling Salesman Problem
- Graph Coloring
- Circuit Satisfiability
- Knapsack
- Subset Sum
- Prime and Factor
- Partition

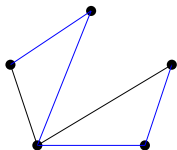
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(Directed) Hamiltonian Cycle and Path

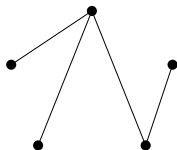
A **Hamiltonian cycle (path)** in graph is a cycle (path) containing all vertices



Hamiltonian cycle in G



No Hamiltonian cycle in G
Hamiltonian path in blue



No Hamiltonian path in G
So no Hamiltonian cycle

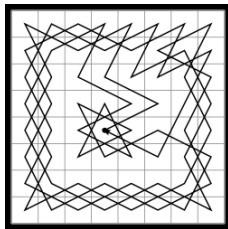
HAM-CYCLE(G) problem: Does G have a Hamiltonian cycle?

HAM-PATH(G) problem: Does G have a Hamiltonian path?

DIR-HAM-CYCLE(G), and **DIR-HAM-PATH(G)** are defined analogously

Hamiltonian Cycle and Path Applications

- Is there a sequence of moves that takes the knight to each square on an 8×8 chessboard exactly once, returning to the original square
- For 8×8 Abu Bakr Muhammad b. Yahya al-Suli found one in 9th century
- For $n \times n$ chessboard define a vertex for each position and connect vertex v_{ij} to vertex v_{kl} if there is a legal move between the (i, j) th position to the (k, l) th position on the board
- Find a Hamiltonian cycle in the graph



Hamiltonian Cycle and Path Applications

Route for School Bus

- Houses considered nodes and streets as edges
- Find a Hamiltonian cycle for a route visiting each student's house exactly once to save fuel and time

Hamiltonian Cycle and Path Applications

Genome Mapping

- Combine many tiny fragments of genetic codes (called “reads”), into one genomic sequence
- Consider each read a node in a graph
- Overlap (end of one read matches the start of another) is an edge
- A Hamiltonian cycle in this graph is a mapping of genome

Longest Path Problem

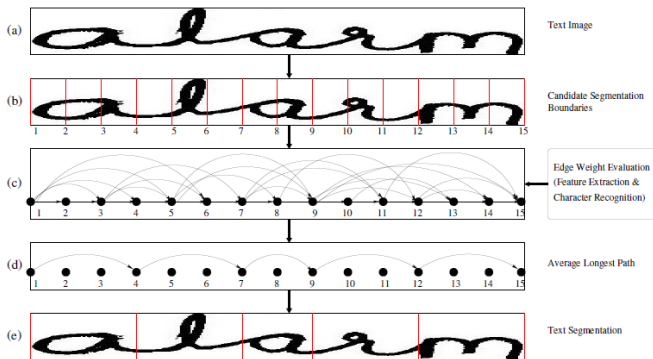
- Given an edge weighted graph $G = (V, E, w)$, two vertices s and t
- The longest $s - t$ path P is a path from s to t with maximum total weight

LONGEST-PATH(G, s, t, k) problem: Is there a $s-t$ path of weight $\geq k$?

Longest Path Problem: Application

Character Segmentation for Optical Character Recognition:

- First step in any OCR system is character segmentation
- Isolate individual characters in hand-written text
- Input to character recognition system
- Salvi et.al. (2013) proposed algorithm based on average longest paths



Static timing analysis (STA)

- Widely used method in circuit design and embedded systems
- STA: a simulation method of computing the expected timing of a digital circuit without requiring to simulate the full circuit
- Longest path identifies critical paths in an IC or VLSI system
- STA is performed only on these critical paths

- **Transportation:** A salesman wants to visit all cities with minimum cost
- **Optimize the tool path** for manufacturing equipment

