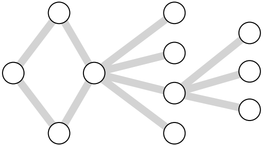
Intractable Problems

- Clique
- Independent Set
- Vertex Cover
- Set Cover
- Set Packing
- Satisfiability Problem
- Hamiltonian Cycle and Path
- Traveling Salesman Problem
- Graph Coloring
- Circuit Satisfiability
- Knapsack
- Subset Sum
- Prime and Factor
- Partition

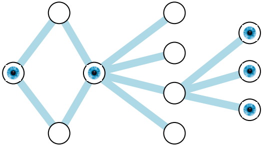
Imdadullah Khan

Vertex Cover

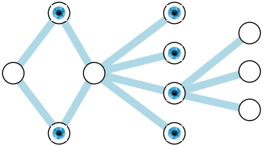
An vertex cover in a graph is subset C of vertices such that each edge has at least one endpoint in C



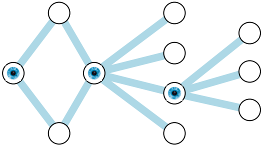
A graph on 11 vertices



A vertex cover of size 5



A vertex cover of size 6



A vertex cover of size 3

The VERTEX-COVER(G, k) problem: Is there a cover of size k in G?

Vertex Cover Application

Network Security: Rout Based Filtering

- Identify a small set of routers/AS
- So as all packets can be monitored at those routers/switches
  - check if the source/destination addresses are valid given the routing table and network topology
- Route-based distributed packet filtering
- Prevents distributed denial of service attacks

Set Cover

- Given a set U of n elements
- A collection $\mathcal{S}$ of m subsets $S_1, S_2, \ldots, S_m \subseteq U$
- A Set Cover is a subcollection $I \subset \{1, 2, \ldots, m\}$ with $\bigcup_{i \in I} S_i = U$

$U := \{1, 2, 3, 4, 5, 6\}$

$\mathcal{S} := \big\{\{1, 2, 3\}, \{3, 4\}, \{1, 3, 4, 5\}, \{2, 4, 6\}, \{1, 3, 5, 6\}, \{1, 2, 4, 5, 6\}\big\}$

Cover-1: $\{1, 2, 3\}, \quad \{1, 3, 4, 5\}, \{2, 4, 6\}$

Cover-2: $\{1, 2, 3\}, \quad \{1, 2, 4, 5, 6\}$

Cover-3: $\{1, 3, 4, 5\}, \quad \{1, 2, 4, 5, 6\}$

Cover-1 has size 3, the latter two have size 2 each

Set Cover

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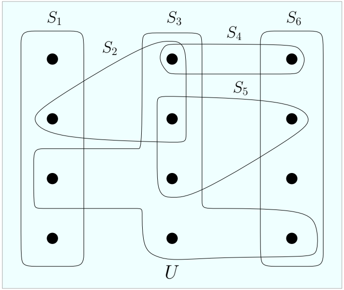
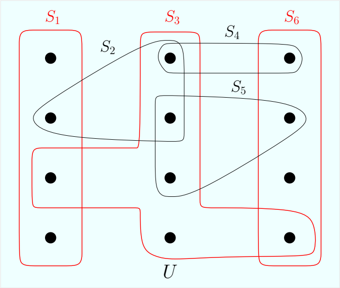




The $\textsc{set-cover}(U, \mathcal{S}, k)$ problem: Is there a cover of size k for U?

Set cover applications

System Integration

- U is the set of capabilities we want our system to have
- $\mathcal{S}$: Available softwares each providing a **subset** of capabilities
- Select a (small) subset of softwares to provide all functionalities

Set cover applications

IBM antivirus tool

- U is the set of (500) known viruses
- They found a set of about 9000 strings of 20 bytes or more that occur in the binaries of viruses but not in "clean" codes
- $\mathcal{S}$: For each string the subset of viruses containing it
- Select a (small) subset of strings that should be searched in codes to detect any virus

Set Packing

- Given a set U of n elements
- A collection $\mathcal{S}$ of m subsets $S_1, S_2, \ldots, S_m \subseteq U$
- A subcollection $I \subset \{1, 2, \ldots, m\}$ pack together if for all $i \neq j \in I$ $S_i \cap S_j = \emptyset$

$U := \{1, 2, 3, 4, 5, 6\}$

$\mathcal{S} := \big\{\{1, 2, 3\}, \{4, 5\}, \{4, 6\}, \{2, 3\}, \{1, 6\}, \{4, 5, 6\}\big\}$

Pack-1: $\{1, 2, 3\}, \{4, 5\}$

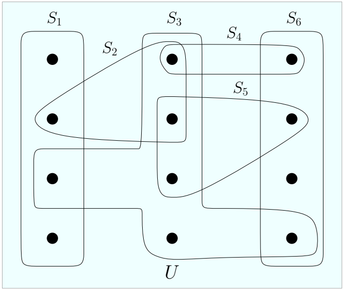
Pack-2: $\{4, 5\}, \{2, 3\}, \{1, 6\}$

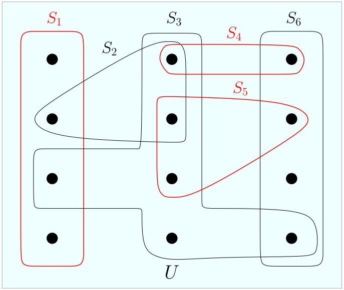
Pack-3: $\{1, 2, 3\}, \{4, 5, 6\}$

Pack-1 and Pack-3 have size 2 each, Pack-3 has size 3

Set Packing

- Given a set U of n elements
- A collection $\mathcal{S}$ of m subsets $S_1, S_2, \ldots, S_m \subseteq U$
- A subcollection $I \subset \{1, 2, \ldots, m\}$ pack together if for all $i \neq j \in I$ $S_i \cap S_j = \emptyset$





The SET-PACKING$(U, \mathcal{S}, k)$ problem: Is there a packing of size k?

Set packing Applications

Resource Sharing

- U is a set of non sharable resources
- $\mathcal{S}$: Processes each requesting a subset of resources
- Select a (large) subset of processes to allocate resources to

Set packing Applications

Airline Crew Scheduling

- U is the set of crew staff
- $\mathcal{S}$: teams of members willing to work with each other
- Select (many) teams to service a set of flights