

## Finite Automata

- Deterministic Finite Automata
- Languages decided by a DFA – Regular Languages
- Closure Properties of regular languages
- Non-Deterministic Finite Automata, DFA= NFA
- Regular Expression: Computation as Description
- DFA=NFA=RegExp, Generalized NFA
- Non-Regular Languages, The Pumping Lemma
- Minimizing DFA

IMDAD ULLAH KHAN

## More Closure Properties of Regular Languages

## Regular Languages are closed under “Reversal”

For every NFA  $N$ , there is a DFA  $M$  such that  $L(M) = L(N)$

Regular Languages = DFA-Recognizable Languages = NFA-Recognizable Languages

### Definition (Regular Language)

A language  $L$  is regular if it is accepted by an NFA

### Theorem

*If  $L$  is a regular language over  $\Sigma$ , then  $L^R$  is also regular*

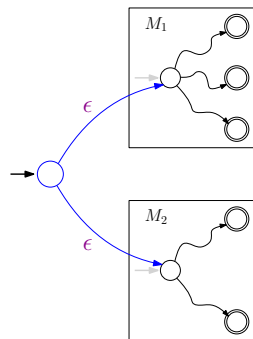
**Proof:** If  $L$  is a regular, then there is a DFA  $M$  recognizing it  
The NFA  $M^R$  recognizes  $L^R$ , thus  $L^R$  is also regular

We can also make a DFA  $M'$  equivalent to  $M^R$

## Closure Properties via NFA

---

NFAs make proving closure properties simpler

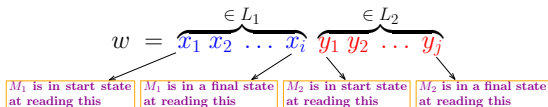


What language is accepted by the new machine?

## Regular Languages are closed under "Concatenation"

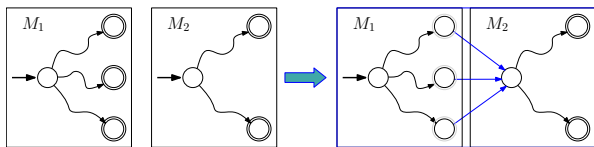
If  $L_1$  and  $L_2$  are regular languages over  $\Sigma$ , then  $L_1 \circ L_2$  is also regular

$M_1$  and  $M_2$ : DFAs recognizing  $L_1$  and  $L_2$ , make  $M$  to recognize  $L_1 \circ L_2$



Intuitively,  $M$  should simulate  $M_1$  until  $x_i$  (check if  $M_1$  is accepting) and then start to simulate  $M_2$  until  $y_j$  (and check if  $M_2$  is accepting)

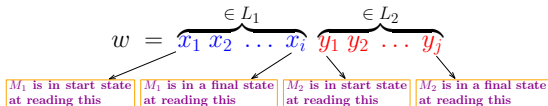
Don't know boundary of  $x$  and  $y$ , attach  $F_1$  to  $q_0^2$



## Regular Languages are closed under "Concatenation"

If  $L_1$  and  $L_2$  are regular languages over  $\Sigma$ , then  $L_1 \circ L_2$  is also regular

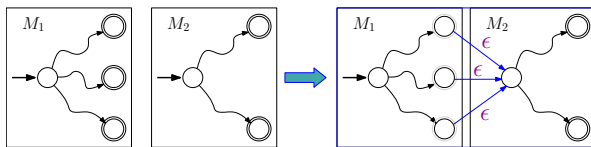
$M_1$  and  $M_2$ : DFAs recognizing  $L_1$  and  $L_2$ , make  $M$  to recognize  $L_1 \circ L_2$



Intuitively,  $M$  should simulate  $M_1$  until  $x_i$  (check if  $M_1$  is accepting) and then start to simulate  $M_2$  until  $y_j$  (and check if  $M_2$  is accepting)

Don't know boundary of  $x$  and  $y$ , attach  $F_1$  to  $q_0^2$

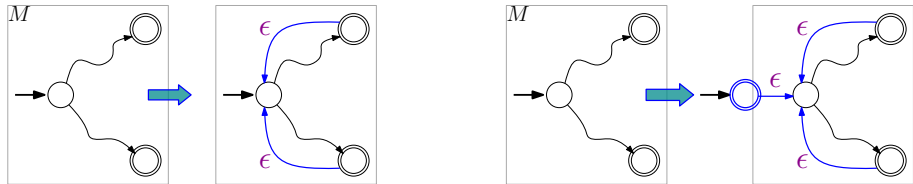
Accepting state of  $M_2$  may be visited many times (don't indicate  $x_i$ ), need to guess end of  $x$  and beginning of  $y$



## Regular Languages are closed under "Star"

$L$  is a regular language  $\implies L^* = \{w_1 \dots w_n : n \geq 0 \wedge w_i \in L\}$  is regular

$M$ : DFAs recognizing  $L$ .  $M^*$  simulates serial cascade of  $M$  to recognize  $L^*$



Give formal construction of NFA's to recognize the concatenation of two regular languages and the star of a regular language