Finite Automata

- Deterministic Finite Automata
- Languages decided by a DFA – Regular Languages
- Closure Properties of regular languages
- Non-Deterministic Finite Automata, DFA= NFA
- Regular Expression: Computation as Description
- DFA=NFA=RegExp, Generalized NFA
- Non-Regular Languages, The Pumping Lemma
- Minimizing DFA

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Language of DFA

DFA: Formal Definition

A DFA M is a 5-tuple $M = (Q, \Sigma, q_0, \delta, F)$

- Q: A finite set of states
- Σ: Alphabet ▷ A finite set of characters
- $q_0 \in Q$ Start or initial state
- $\delta : Q \times \Sigma \mapsto Q$ Transition function
- $F \subseteq Q$ Set of accept/final states

M accepts $w = w_1, w_2, \ldots, w_n \in \Sigma^n$

if there is a sequence of states $r_0, r_1, \ldots, r_n$ such that

$r_0 = q_0, \quad \delta(r_i, w_{i+1}) = r_{i+1}, \quad r_n \in F$

Decision Problem = Language Recognition Problem

There is a one-to-one correspondence between decision problem and language recognition problem

Every language L over Σ uniquely corresponds to a decision problem $f : \Sigma^* \mapsto \{\textbf{Yes}, \textbf{No}\}$

$$L \ = \ \{w \ : \ f(w) = \textbf{Yes}\}$$

A decision problem is the task of recognizing whether a given string (instance) is in a language

Language Decided by DFA

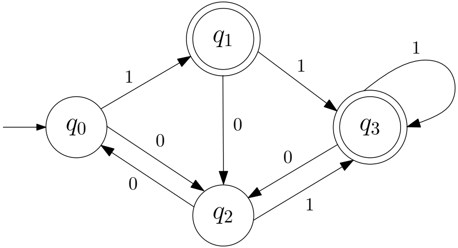
Let M be a DFA over Σ

$$L(M) = \{w \in \Sigma^* : M \text{ accepts } w\}$$

$L(M)$ is the language decided (or accepted/recognized/computed) by M

Language Decided by DFA

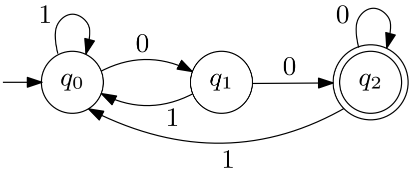
What language is decided by the following DFA (in the obvious Σ)?



$L(M) = \{w : w \text{ ends with a } 1\}$

Language Decided by DFA

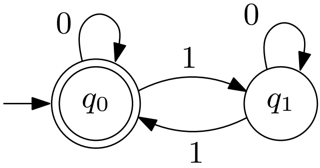
What language is decided by the following DFA (in the obvious Σ)?



$L(M) = \{w : w \text{ ends the substring } 00\}$

Language Decided by DFA

What language is decided by the following DFA (in the obvious Σ)?

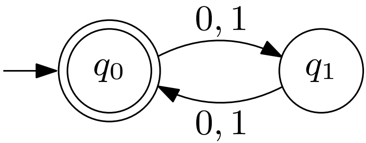


$L(M) = \{w : w \text{ contains an even number of 1's}\}$

Give a formal proof by induction (on the length of strings)

Language Decided by DFA

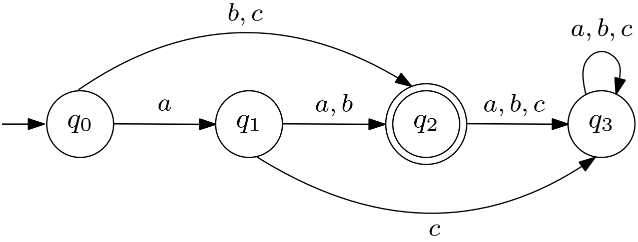
What language is decided by the following DFA (in the obvious Σ)?



$L(M) = \{w : w \text{ has even length}\}$

Language Decided by DFA

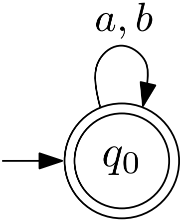
What language is decided by the following DFA (in the obvious Σ)?



$L(M) = \{b, c, aa, ab\}$

Language Decided by DFA

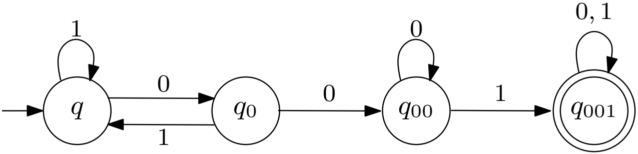
What language is decided by the following DFA (in the obvious Σ)?



$L(M) = \{a, b\}^*$

Language Decided by DFA

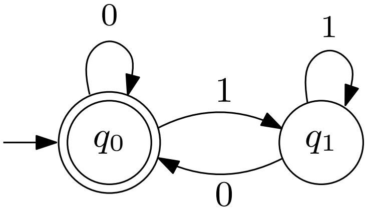
What language is decided by the following DFA (in the obvious Σ)?



$L(M) = \{w : w \text{ contains the substring } 001\}$

Language Decided by DFA

What language is decided by the following DFA (in the obvious Σ)?

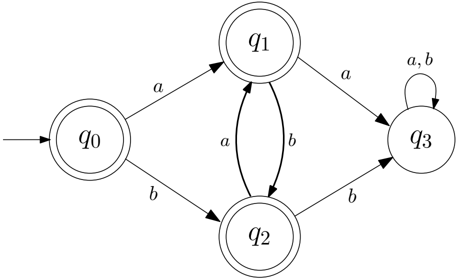


$L(M) = \{w : w \text{ ends with a } 0\} \cup \{\epsilon\}$

ϵ is the unique string of length 0, the empty string

Language Decided by DFA

What language is decided by the following DFA (in the obvious Σ)?



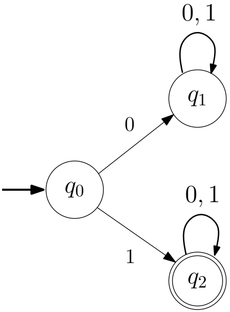
$L(M) = \{w : w \text{ does not contain } aa \text{ or } bb\}$

q_3 serves as a trap state

DFA for Languages

Design a DFA (with as few states as you can) for the following language

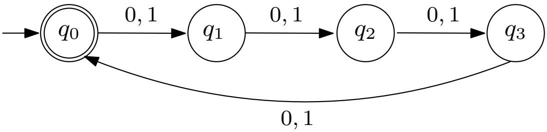
$$L_1 \subseteq \{0,1\}^* = \{w : w \text{ begins with a } 1\}$$



DFA for Languages

Design a DFA (with as few states as you can) for the following language

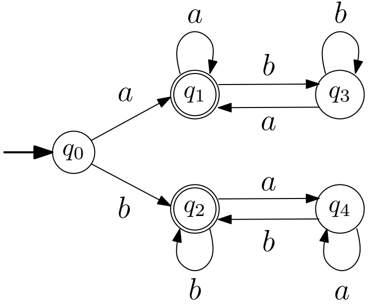
$$L_2 \subseteq \{0,1\}^* = \{w : |w| \text{ is divisible by } 4\}$$



DFA for Languages

Design a DFA (with as few states as you can) for the following language

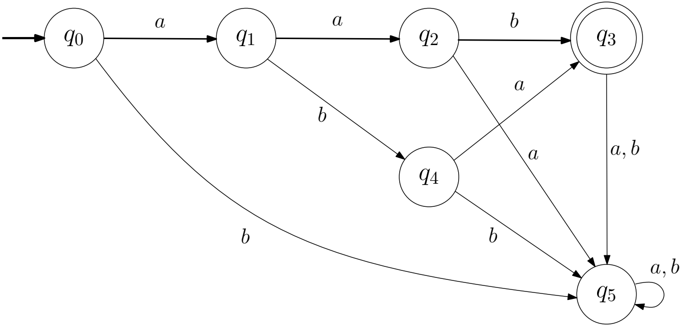
$$L_3 \subseteq \{a, b\}^* \; = \; \{w : w \text{ starts and ends with the same character}\}$$



DFA for Languages

Design a DFA (with as few states as you can) for the following language

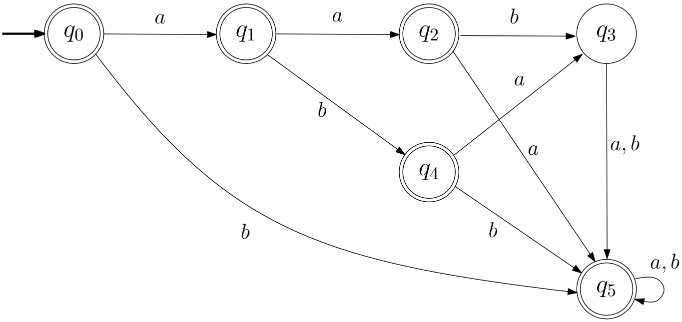
$$L_1 \subseteq \{a, b\}^* \; = \; \{aab, aba\}$$



DFA for Languages

Design a DFA (with as few states as you can) for the following language

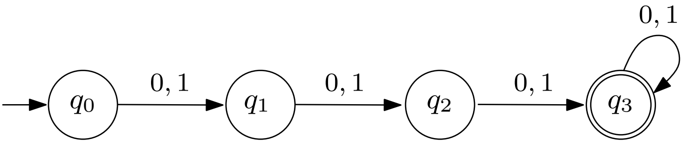
$$\overline{L_1} \subseteq \{a, b\}^* \; = \; \overline{\{aab, aba\}}$$



DFA for Languages

Design a DFA (with as few states as you can) for the following language

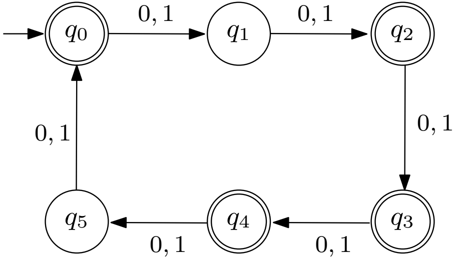
$$L_4 \subseteq \{0,1\}^* = \{w : w \text{ has length at least } 3\}$$



DFA for Languages

Design a DFA (with as few states as you can) for the following language

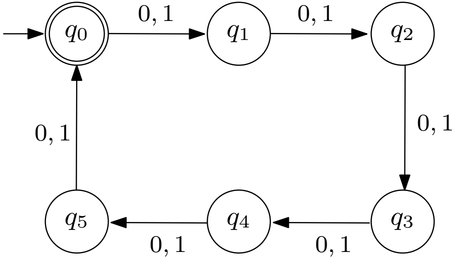
$$L_6 \subseteq \{0,1\}^* = \{w : w \text{ has length divisible 2 } \textsc{or} \text{ 3}\}$$



DFA for Languages

Design a DFA (with as few states as you can) for the following language

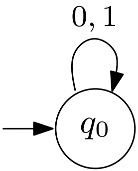
$$L_6 \subseteq \{0, 1\}^* \; = \; \{w : w \text{ has length divisible 2 AND 3}\}$$



DFA for Languages

Design a DFA (with as few states as you can) for the following language

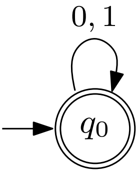
$$L_6 \subseteq \{0,1\}^* \;=\; \emptyset$$



DFA for Languages

Design a DFA (with as few states as you can) for the following language

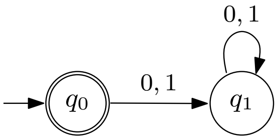
$$L_6 = \{0,1\}^*$$



DFA for Languages

Design a DFA (with as few states as you can) for the following language

$$L_6 \subseteq \{0,1\}^* = \{\epsilon\} = \{w : w \text{ is the empty string}\}$$



DFA for Languages

Design a DFA (with as few states as you can) for the following languages

- $L_5 \subseteq \{0,1\}^* \;=\; \{w : w \text{ has length divisible } 3\}$
- $L_7 \subseteq \{0,1\}^* \;=\; \{w : w \text{ has length divisible } 2 \text{ AND } 3\}$
- $L_{10} \subseteq \{0,1\}^* \;=\; \{\epsilon, 001, 001001, 001001001, \ldots, \}$
- $L_{11} \subseteq \{0,1\}^* \;=\; \{w : w \text{ contains the substring } 101\}$