

## Finite Automata

- Deterministic Finite Automata
- Languages decided by a DFA – Regular Languages
- Closure Properties of regular languages
- Non-Deterministic Finite Automata, DFA= NFA
- Regular Expression: Computation as Description
- DFA=NFA=RegExp, Generalized NFA
- Non-Regular Languages, The Pumping Lemma
- Minimizing DFA

IMDAD ULLAH KHAN

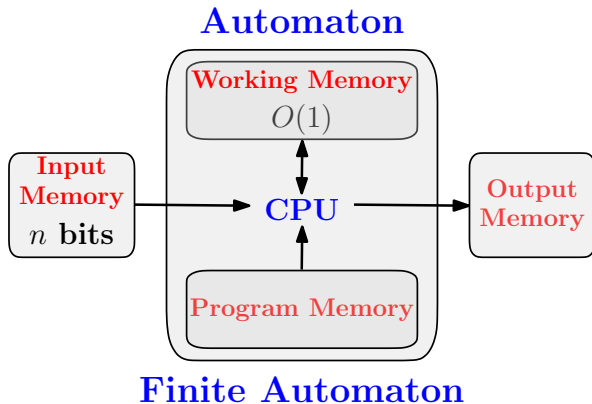
# Deterministic Finite Automata

## Models of Computation: Finite Automata

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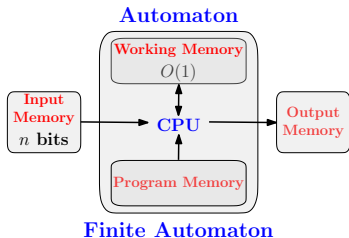
Automata are distinguished by type/amount of working memory

A **D**eterministic **F**inite **A**utomata has constant working memory



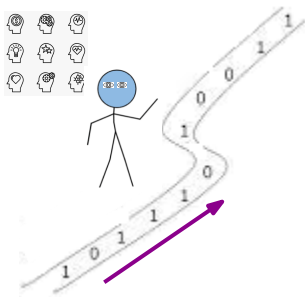
## Anatomy of DFA

A **D**eterministic **F**inite **A**utomata has constant working memory



A deterministic finite automaton or a **finite state machine** is a little creature

- it has tiny eyes – sees one symbol
- changes its state of mind according to the symbol it sees
- only remember its current state of mind

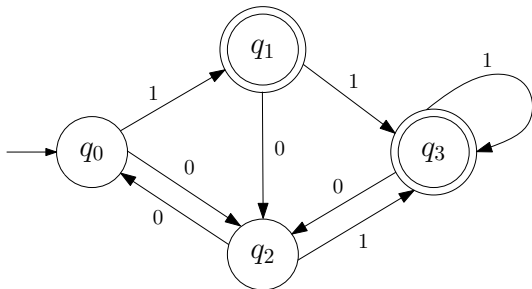


## Anatomy of DFA

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A DFA over alphabet  $\Sigma$  is depicted as a directed graph with self-loops

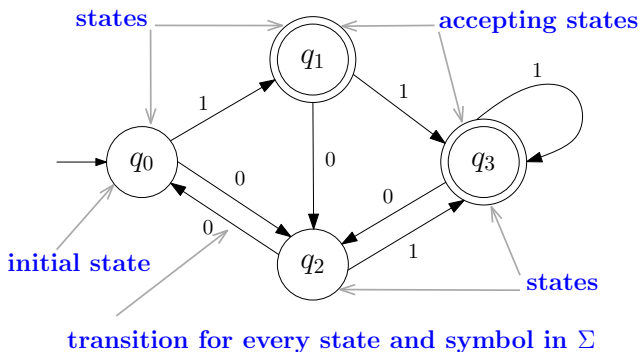
▷ called state diagram of the DFA



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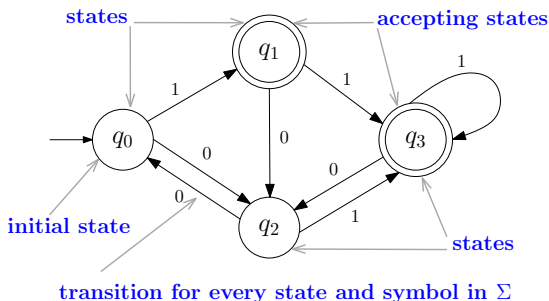


## Simulation of DFA

DFA begins in the (unique) initial state and read the input left-to-right one character at a time

It transition to the next state according to transition rules (labeled edges)

The automaton **accepts** the input string if the last state is an accepting state (double-circled). Else, it **rejects** the input string.

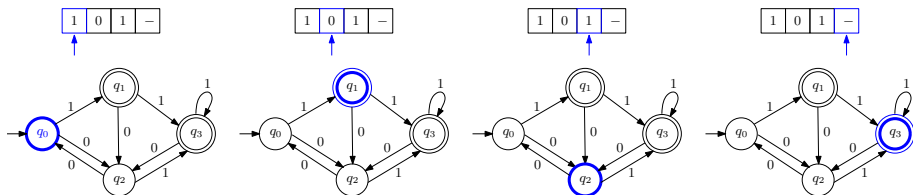


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This DFA accepts the input string 101

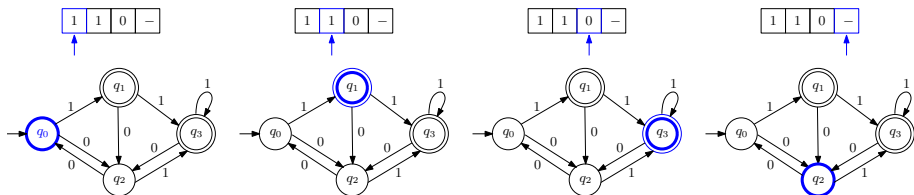


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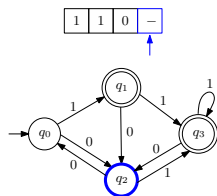
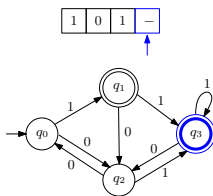
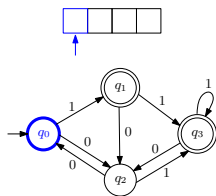
This DFA rejects the input string 110

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Which strings are accepted by this DFA?

## DFA: Formal Definition

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A DFA  $M$  is a 5-tuple  $M = (Q, \Sigma, q_0, \delta, F)$

- $Q$ : A finite set of states
- $\Sigma$ : Alphabet ▷ A finite set of characters
- $q_0 \in Q$  Start or initial state
- $\delta : Q \times \Sigma \mapsto Q$  Transition function
- $F \subseteq Q$  Set of accept/final states

$M$  accepts  $w = w_1, w_2, \dots, w_n \in \Sigma^n$

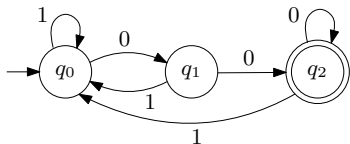
if there is a sequence of states  $r_0, r_1, \dots, r_n$  such that

$$r_0 = q_0, \quad \delta(r_i, w_{i+1}) = r_{i+1}, \quad r_n \in F$$

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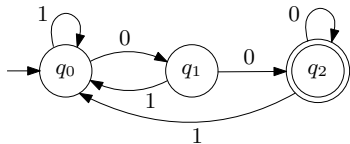
- $Q = \{q_0, q_1, q_2\}$
- $\Sigma = \{0, 1\}$
- Initial State:  $q_0$ 
  - $\delta(q_0, 0) = q_1, \quad \delta(q_0, 1) = q_0$
  - $\delta(q_1, 0) = q_2, \quad \delta(q_1, 1) = q_0$
  - $\delta(q_2, 0) = q_2, \quad \delta(q_2, 1) = q_0$
- $F = \{q_2\}$

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Transition function  $\delta$  can be depicted in a transition table (lookup table)



	0	1
$q_0$	$q_1$	$q_0$
$q_1$	$q_2$	$q_0$
$q_2$	$q_2$	$q_0$

## DFA as Programming Code

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### Algorithm DFA as Programming Code

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```
STATE  $\leftarrow q_0$ 
 $i \leftarrow 0$ 
while  $input[i] \neq EOF$  do
    STATE  $\leftarrow \delta(\text{STATE}, input[i])$ 
     $i \leftarrow i + 1$ 
if STATE  $\in F$  then
    return Accept
else
    return Reject
```