# Computability Theory: Decidability and Recognizability

- Encoding Turing Machines and the Universal TM
- Computability
- Halt: Undecidable Problems using Diagnolization
- Accept: Undecidable Problems using Diagnolization
- Turing Reductions
- Mapping Reductions
- Undecidable and Unrecognizable Problems
- Rice Theorem

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# Non-TM Computability Questions

## Non-TM Computability Questions

Not all undecidable and unrecognizable languages involve TM

- Hilbert's 10th problem:

  Given a diophantine equation with integer coefficients have integral solution

  ▷ undecidable (Matiyasevich, Robinson, Davis, Putnam 1970)

- Does it have a real root?

  ▷ decidable (Tarski 1951)

- Does it have a rational root?

  ▷ is not known to be decidable or not

# Non-TM Computability Questions

Not all undecidable and unrecognizable languages involve TM

Mortal Matrices: Given two $n \times n$ matrices $A$ and $B$. Can we multiply $A$ and $B$ together as many times as we want so we get the 0 matrix?

▷ undecidable for $n = 3, 15, 21$ (Halava, Jarju, Hirvensalo, 2007)

$$A = \begin{bmatrix} 2 & 0 \\ 3 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 5 \end{bmatrix}, \quad AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad BA = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Not all undecidable and unrecognizable languages involve TM

Richardson's Problem: Given a set of rational numbers $S$. Let $E$ be a function in one variable, constructed using the numbers in $S \cup \{\pi, \ln(2)\}$ and the variable $x$ combined using operations, $+$, $-$, $*$, sin, exp, abs.

Given such an expression $E(x)$ check if $E(x) \ = \ 0$

$\triangleright$ undecidable (Richardson, 1868)