Computability Theory: Decidability and Recognizability

- Encoding Turing Machines and the Universal TM
- Computability
- Halt: Undecidable Problems using Diagnolization
- Accept: Undecidable Problems using Diagnolization
- Turing Reductions
- Mapping Reductions
- Undecidable and Unrecognizable Problems
- Rice Theorem

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An unrecognizable problem

The complement of A_{TM}

$A_{TM} = \{\langle M, w\rangle : M \text{ a TM over } \Sigma,\ M \text{ accepts } w \in \Sigma^*\}$

$A_{TM} \subset \{0,1\}^*$, What is its complement $\overline{A_{TM}}$?

If $x \in \{0,1\}^*$ be a string.

If $x \notin A_{TM}$, then can we say the machine does not accept the string?

$\triangleright$ What machine? what string?

We define the decoding function as follows:

If $x \in \{0,1\}^*$ does not decode to a pair $\langle M, w\rangle$, then we say that x decodes to the pair $\langle D_M, \epsilon\rangle$, where D_M is a dummy Turing machine that accepts no string.

With this decoder we can say that

$\overline{A_{TM}} = \{\langle M, w\rangle : M \text{ a TM over } \Sigma,\ M \text{ does not accept } w \in \Sigma^*\}$

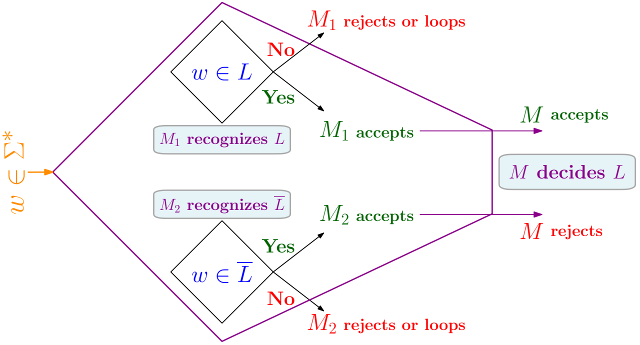
A concrete unrecognizable problem

Earlier, we showed that there exists unrecognizable problems

▷ number of Turing machines is less than number of languages

Can we give a concrete example of an unrecognizable problem?

$L \subset \Sigma^*$: a language. If both L and $\overline{L}$ are recognizable, then L is decidable



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1. $A_{TM} = \{\langle M, w\rangle : M$ a TM over $\Sigma,\ M$ accepts $w \in \Sigma^*\}$ is recognizable
2. $A_{TM} = \{\langle M, w\rangle : M$ a TM over $\Sigma,\ M$ accepts $w \in \Sigma^*\}$ is undecidable

$\overline{A_{TM}} = \{\langle M, w\rangle : M$ a TM over $\Sigma,\ M$ does not accept $w \in \Sigma^*\}$ is unrecognizable