Computability Theory: Decidability and Recognizability

- Encoding Turing Machines and the Universal TM
- Computability
- Undecidable Problems using Diagnolization
- The Halting and Accept Problems
- Turing Reductions
- Mapping Reductions
- Undecidable and Unrecognizable Problems
- Rice Theorem

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Encoding Turing Machines and the Universal TM

Reprogramming Turing Machines

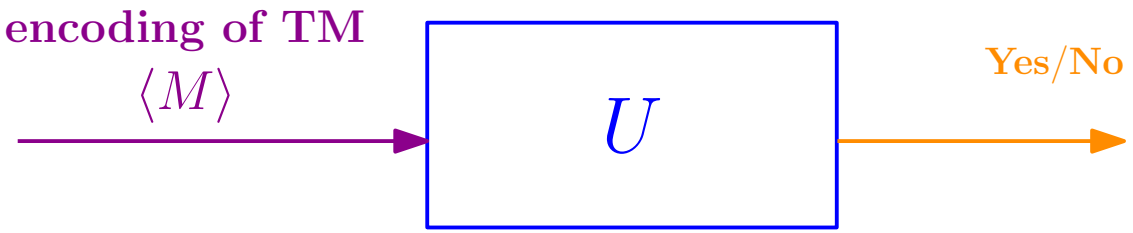
Can Turing Machines be re-programmed?

Computers are "general purpose" machines – they are programmable (can execute other programs)

While a Turing Machine executes a fixed program – they are hard-wired (not programmable)

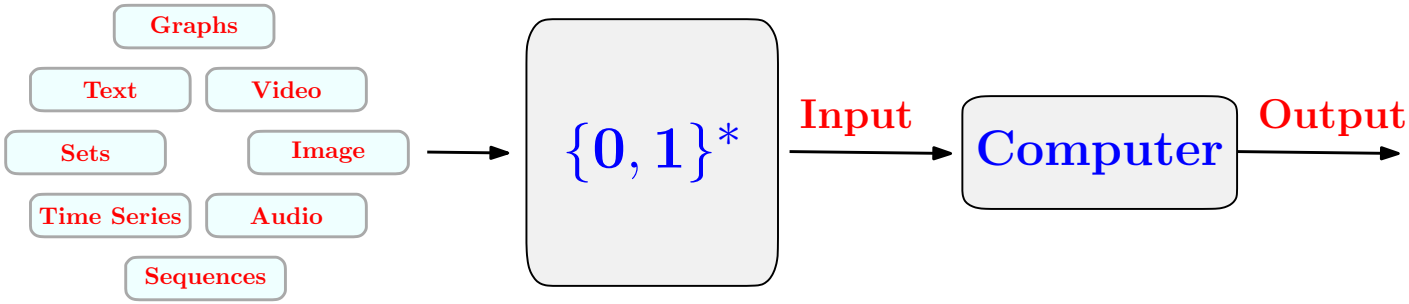
A universal Turing Machine U can analyze and simulate other TMs

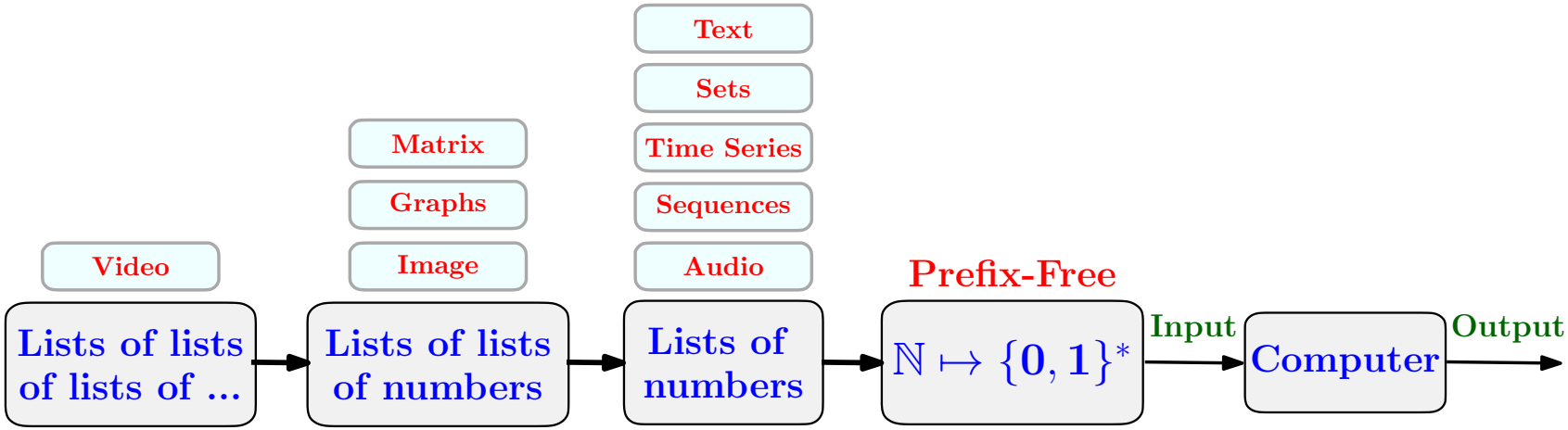
Need a way to encode Turing Machines (like C++/Java/Python encodes computer programs)



Encoding Turing Machines

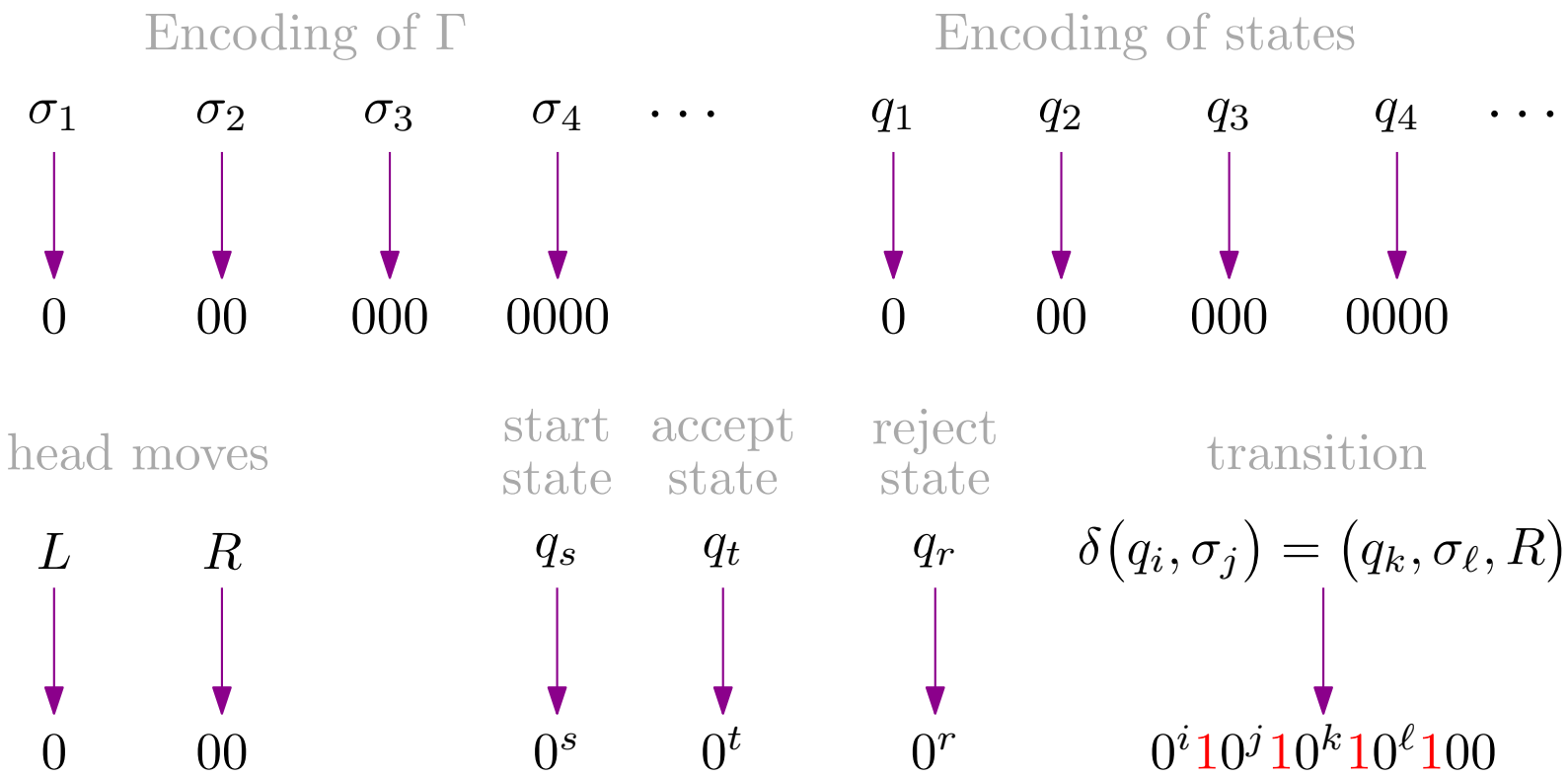
We gave binary representation for common types of data





Encoding Turing Machines

Encoding Turing Machine as bit strings



Encoding of Turing machine

Q **1** Σ **1** Γ **1** q_s **1** q_t **1** q_r **1** transition1 **1** transition2 **1** ...

1 : separator

Encoding Turing Machines

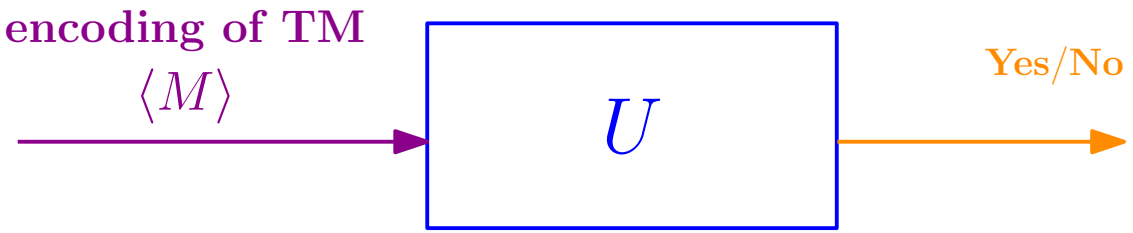
Any subset of Turing machines is a binary language

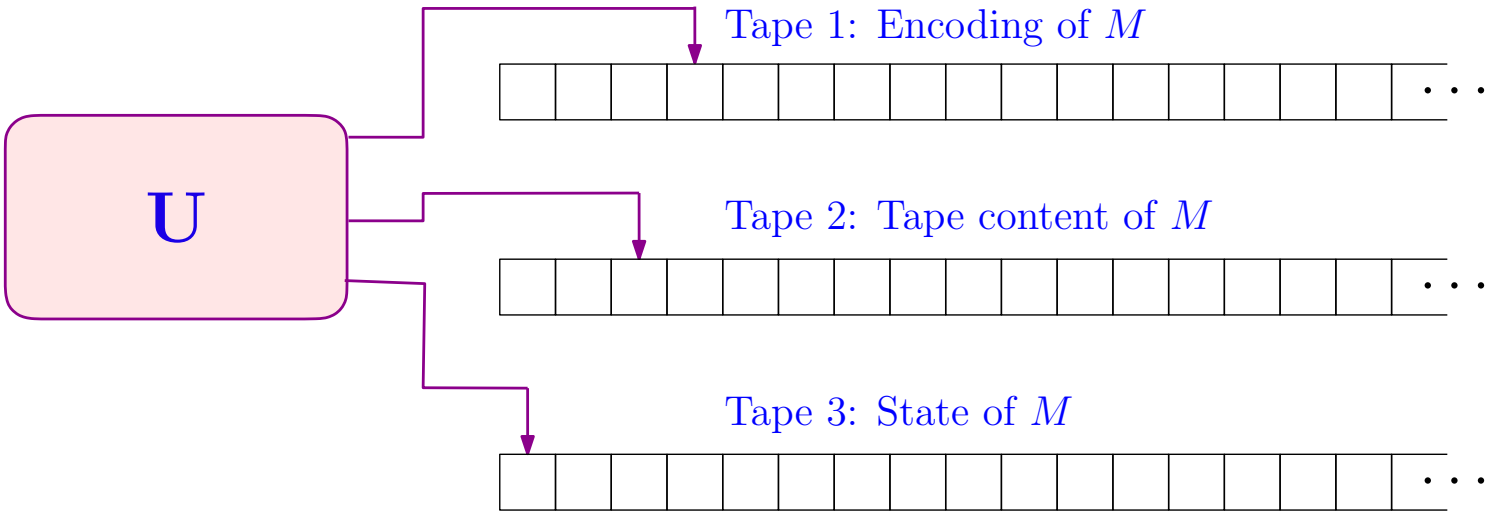
We gave an encoding of Turing machines as bit strings, thus a TM $M \in \{0,1\}^*$

$\therefore$ any set of Turing machines (a set of bit strings) is a subset of $\{0,1\}^*$

The set of Turing machines is a language over $\{0,1\}$ (each string is a binary encoding of a Turing machine)

Universal Turing Machine





Universal Turing Machine

Theorem

There is a Turing machine U that takes as input an encoding of an arbitrary Turing machine M over Σ and a string $w \in \Sigma^*$ such that U accepts $\langle M, w \rangle$ if and only if M accepts w

Existence of a universal Turing machine is a fundamental property of Turing machines

DFA/NFA do not have this property