

Turing Machines

- Turing Machine: Model of Computation
- Turing Machine: Anatomy and Working
- Turing Machine: Formal Definition and Rules of Computation
- Recognizable and Decidable Languages
- Turing Machine: Levels of Abstraction
- Variants of Turing Machine and The Church-Turing Thesis
- Non-Deterministic Turing Machine

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Turing Machine: Three Levels of Abstraction

Three Levels of Abstraction in Describing Turing Machines

- **Low Level Description**

Explicitly describe the finite state control, all states and transitions

- **Intermediate Level Description**

Describe in English the finite state control, state transition, writing on tape, and head movements

Should be readily translatable into a Low Level Description

- **High Level Description**

Give an algorithm in pseudocode or English

Skips standard details, just highlight the main idea of solution

Turing Machine: High Level description

High Level description of TM to decide $L = \{0^{2^n} : n \geq 0\}$

Algorithm Powers of $2(w)$

▷ check if $|w| = 2^n$

```
1: while true do  
2:   if  $|w| = 1$  then  
3:     return Accept  
4:   else if  $|w|$  is odd then  
5:     return Reject  
6:   else  
7:     delete half of the 0's in  $w$ 
```

In every iteration, the number of 0's on the tape is halved. The string is accepted if and only if the number of 0's is a power of 2.

Turing Machine: Medium Level description

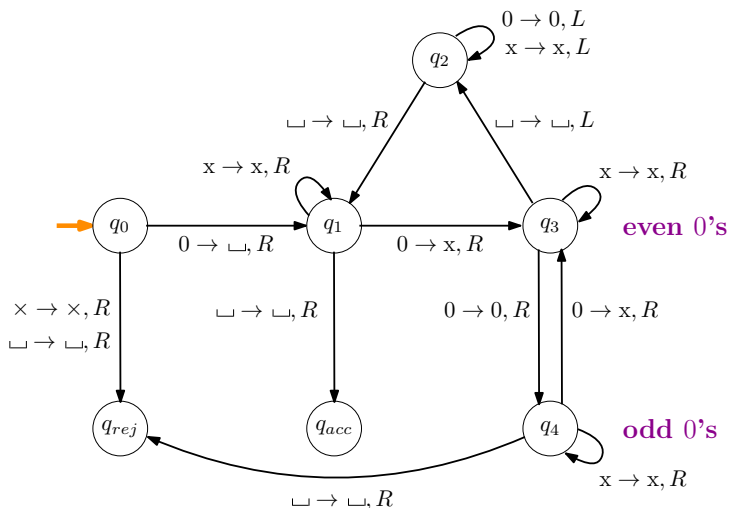
Medium Level description of TM to decide $L = \{0^{2^n} : n \geq 0\}$

- 1 Move the head from left to right, cross out every other 0
 - a If in step 1 there is only one 0, **accept**
 - b If in step 1 there is an odd (> 1) number of 0's **reject**
- 2 Return the head back to the left end of the tape
- 3 Go back to step 1

Again, in every iteration, the number of 0's on the tape is halved. The string is accepted if and only if the number of 0's is a power of 2.

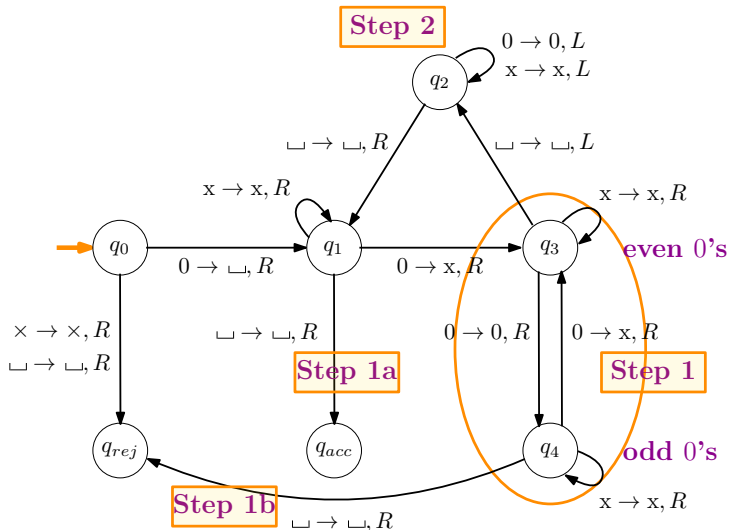
Turing Machine: Low Level description

Low Level description of TM to decide $L = \{0^{2^n} : n \geq 0\}$



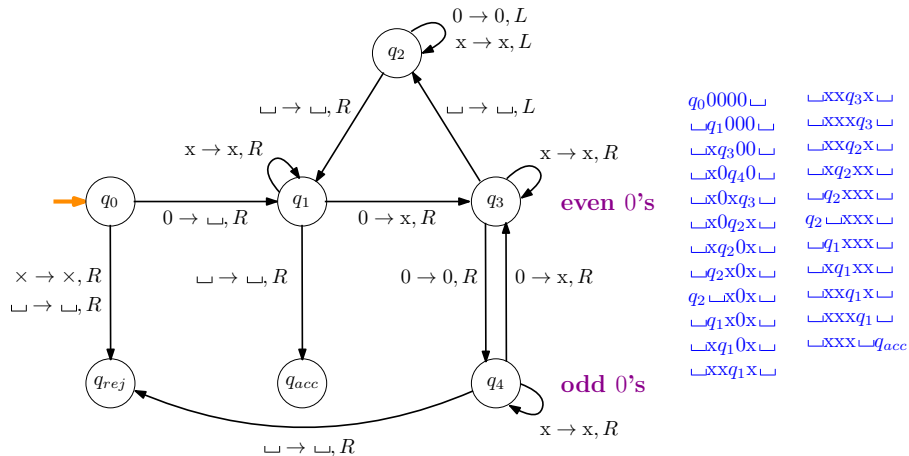
Turing Machine: Low Level description

Low Level description of TM to decide $L = \{0^{2^n} : n \geq 0\}$



Turing Machine: Low Level description

Low Level description of TM to decide $L = \{0^{2^n} : n \geq 0\}$



Run the TM on $\epsilon, 0, 00, 000$

Turing Machine: High Level description

High Level description of TM to decide $L = \{a^n b^n : n \geq 0\}$

Algorithm check if $w \in L = a^n b^n$

- 1: **while true do**
 - 2: **if** $|w| = 0$ **then**
 - 3: **return Accept**
 - 4: **else**
 - 5: delete an a and a b from first and second half of w respectively
-

In every iteration, the number of a 's and b 's in each half is reduced by 1.

The string is accepted if and only if the first half is all and only a 's and the second half is all and only b 's

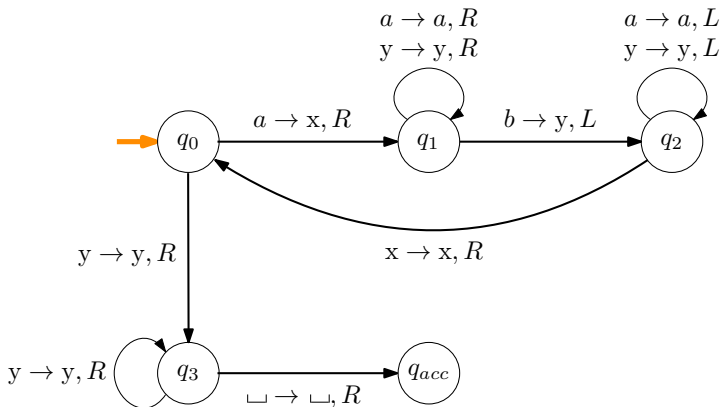
Turing Machine: Medium Level description

Medium Level description of TM to decide $L = \{a^n b^n : n \geq 0\}$

- 1 Mark the first a as x and move head to the first b and mark it as y
- 2 Move the head left to the second a and mark it x and move the head to second b and mark it y
- 3 Repeat until all a 's and b 's are replaced with x 's and y 's
- 4 Move the head from left most to right most symbol and check if all and only x 's precede all and only y 's
▷ No need to count now

Turing Machine: Low Level description

Low Level description of TM to decide $L = \{a^n b^n : n \geq 0\}$



Run the machine on $aabb$, ϵ , $aaaabbb$, $aabbb$