Turing Machines

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Turing Machine: Formal Definition and Rules of Computation

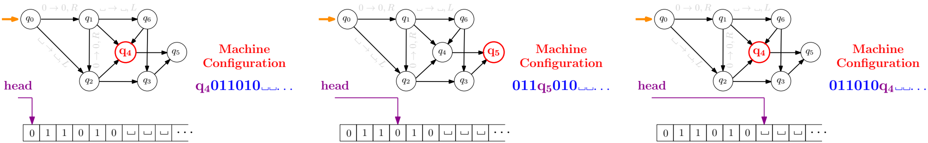
Turing Machine: Formal Definition

A Turing machine is a 7-tuple $T = (Q, \Sigma, \Gamma, q_0, q_{accept}, q_{reject}, \delta)$

- Q: a finite set of states
- Σ: input alphabet $\sqcup \notin \Sigma$
- Γ: tape alphabet, $\Sigma \subseteq \Sigma$ and $\sqcup \in \Gamma$
- $q_{accept} \in Q$: is the accept state
- $q_{reject} \in Q$: is the reject state, $q_{reject} \neq q_{accept}$
- $\delta : Q \setminus \{q_{accept}, q_{reject}\} \times \Gamma \mapsto Q \times \Gamma \times \{L, R\}$: transition function
- $q_0 \in Q$: start state

Turing Machine: Configuration

A configuration is a representation of the current state of the Turing machine, the content of the tape, and the current location of the head



Formally, a configuration is a string uqv, where $u, v \in \Gamma^*$, $q \in Q$

- Current state is q
- Tape contents is uv followed by infinitely many $\sqcup$
- Current head location is the first symbol in v

A Turing machine is a 7-tuple $T = (Q, \Sigma, \Gamma, q_0, q_{accept}, q_{reject}, \delta)$

Tape starts with input $w \in \Sigma^*$, followed by infinite $\sqcup$'s

Head is initially on the leftmost cell

Control starts in q_0

If current state is q and head is on a cell containing symbol $\sigma \in \Gamma$, T transitions according to $\delta(q_i, \sigma_i) = (q_j, \sigma_j, dir \in \{L, R\})$, where

- q_j is the next state (q_j could be $= q_i$)
- σ_j is the symbol written on the current cell (σ_j could be $= \sigma_i$)
- $dir = L/R \implies$ the head move to the cell on left/right
  - ▷ head is on leftmost cell and $dir = L \implies$ head stays on same cell

No transitions from q_{accept} or q_{reject}

T halts when q_{accept} or q_{reject} is reached or there is no possible transition

T might loop forever

Turing Machine: Rules of Computation

A Turing machine is a 7-tuple $T = (Q, \Sigma, \Gamma, q_0, q_{accept}, q_{reject}, \delta)$

On input $w \in \Sigma^*$, initial configuration $q_0 w$

- If $\delta(q, b) = (q', c, R)$, then configuration $uaqbv$ yields $uacq'v$

  $\triangleright$ $uaqbv \succ uacq'v$

- If $\delta(q, b) = (q', c, L)$, then configuration $uaqbv$ yields $uq'acv$

  $\triangleright$ $uaqbv \succ uq'acv'v$

- If $\delta(q, b) = (q', c, L)$, then configuration qbv yields $q'cv$

  $\triangleright$ $qbv \succ q'cv$

Accepting configuration: $u\, q_{accept}\, v$

Rejecting configuration: $u\, q_{reject}\, v$

Turing Machine: Acceptance and Rejection

A TM $M = (Q, \Sigma, \Gamma, q_0, q_{accept}, q_{reject}, \delta)$ accepts a string $w \in \Sigma^*$

if there is a sequence of configurations $C_0, C_2, \ldots, C_k$ such that

- $C_0 = q_0 w$
- $C_i \succ C_{i+1}$ for $i = 0, 1 \ldots, k-1$
- C_k is an accepting configuration i.e. C_k contains q_{accept}

We say that $C_0 \succ^* C_k$

We refer to $C_0, C_2, \ldots, C_k$ as history or trace of M on w