Computation, Encoding and Languages

- Computational Problems, Strings and Data Encoding
- Binary Encoding
- Language
- Versions of Computational Problems
- Decision Problems as Language Recognition
- Models of Computation – CPU + Memory

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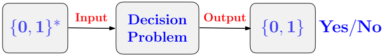
Computational Problems as Languages

Decision Problem = Language Recognition Problem

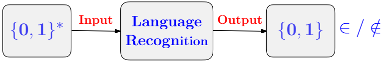
A decision problem is characterized by three things

- $\mathcal{I}$: set of (valid) input instances $\subseteq \{0,1\}^*$
- S: solution space, $\{\textbf{Yes}, \textbf{No}\} = \{0,1\}$
- $f : \mathcal{I} \rightarrow \{0,1\}$: The computational question or function



The language recognition problem is characterized by three things

- $\mathcal{U}$: Universal Set $\subseteq \{0,1\}^*$
- S: Membership Decision, $\{\textbf{Yes}, \textbf{No}\} = \{0,1\}$
- $f : \mathcal{I} \rightarrow \{0,1\}$: Membership Predicate



Decision Problem = Language Recognition Problem

There is a one-to-one correspondence between decision problems and language recognition problems

Every language L over Σ uniquely corresponds to a decision problem $f : \Sigma^* \mapsto \{\textbf{Yes}, \textbf{No}\}$

$$L = \left\{ w : f(w) = \textbf{Yes} \right\}$$

A decision problem is the task of recognizing whether a given string (instance) is in a language

Example of computational problem as Language

- Parity: Does a given string in $\{a, b\}^*$ contain an even number of a's

$$L_1 = \{w : \; w \text{ has an even number of } a\text{'s}\}$$

- PRIME: Is a given $x \in \mathbb{N}$ (in binary representation) a prime number

$$L_2 = \{x : \; x \text{ is a prime number}\}$$

- Halting Problem: Does a given C program ever halt, $\Sigma = \text{ASCII}$,

$$L_3 = \{X.cpp : \; X \text{ halts}\}$$

- $L_2 = \{2, 3, 5, 7, 11, \ldots\}$
- $L_2 = \{11, 111, 11111, 1111111, \ldots\}$
- $L_2 = \{10, 11, 101, 111, 1011, \ldots\}$

The correct answer depends on Σ

Example of computational problem as Language

$\Sigma = \{a, b\}$

Algorithm A Language Recognizer

$w \leftarrow x_1x_2x_3 \ldots x_n$ $\quad \triangleright\ x_i \in \{a, b\}$

$count \leftarrow 0$

for $i = 1 \rightarrow n$ **do**

  if $x_i = a$ **then**

    $count \leftarrow count + 1$

if $count \leq 4$ **then**

  Accept

else

  Reject

What language does this program accept/recognize?

1. $\{w \in \{a, b\}^* : |w| \leq 4\}$
2. $\{w \in \{a, b\}^* : |w| = 4\}$
3. $\{w \in \{a, b\}^* : |w| \geq 4\}$
4. $\{w \in \{a, b\}^* :$ w has at least 4 a's $\}$
5. $\{w \in \{a, b\}^* :$ w has at most 4 a's $\}$
6. $\{w \in \{a, b\}^* :$ w has at exactly 4 a's $\}$