

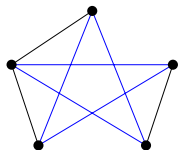
Intractable Problems

- Clique
- Independent Set
- Vertex Cover
- Set Cover
- Set Packing
- Satisfiability Problem
- Hamiltonian Cycle and Path
- Traveling Salesman Problem
- Graph Coloring
- Circuit Satisfiability
- Knapsack
- Subset Sum
- Prime and Factor
- Partition

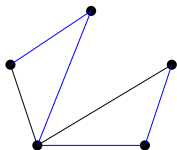
IMDAD ULLAH KHAN

(Directed) Hamiltonian Cycle and Path

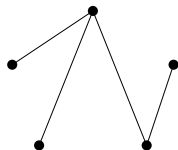
A **Hamiltonian cycle (path)** in graph is a cycle (path) containing all vertices



Hamiltonian cycle in G



No Hamiltonian cycle in G
Hamiltonian path in blue



No Hamiltonian path in G
So no Hamiltonian cycle

HAM-CYCLE(G) problem: Does G have a Hamiltonian cycle?

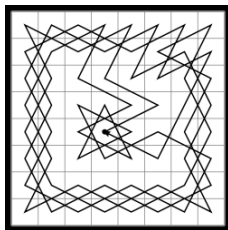
HAM-PATH(G) problem: Does G have a Hamiltonian path?

DIR-HAM-CYCLE(G), and **DIR-HAM-PATH(G)** are defined analogously

Hamiltonian Cycle and Path Applications

Is there a sequence of moves that takes the knight to each square on an 8×8 chessboard exactly once, returning to the original square?

- For 8×8 Abu Bakr Muhammad b. Yahya al-Suli found one in 9th century
- For $n \times n$ chessboard define a vertex for each position and connect vertex v_{ij} to vertex v_{kl} if there is a legal move between the (i, j) th position to the (k, l) th position on the board
- Find a Hamiltonian cycle in the graph



Hamiltonian Cycle and Path Applications

Route for School Bus

School bus should visit each house exactly once to save fuel and time

- Houses considered nodes and streets as edges
- Find a Hamiltonian cycle

Hamiltonian Cycle and Path Applications

Genome Mapping

Combine many tiny fragments of genetic codes (called “reads”), into one genomic sequence

- Consider each read a node in a graph
- Overlap (end of one read matches the start of another) is an edge
- Find a Hamiltonian cycle in this graph, a mapping of genome

Longest Path Problem

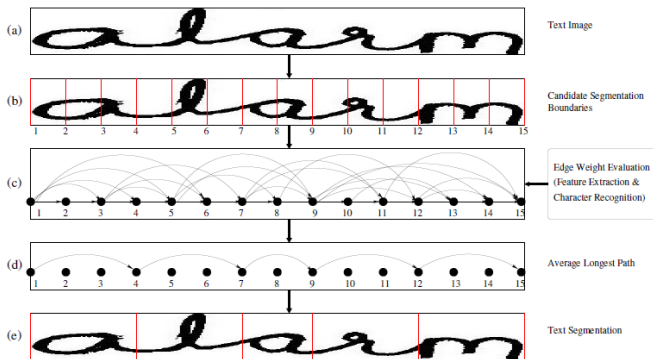
- Given an edge weighted graph $G = (V, E, w)$, two vertices s and t
- The longest $s - t$ path P is a path from s to t with maximum total weight

LONGEST-PATH(G, s, t, k) problem: Is there a $s-t$ path of weight $\geq k$?

Longest Path Problem: Application

Character Segmentation for Optical Character Recognition:

- First step in any OCR system is character segmentation
- Isolate individual characters in hand-written text
- Input to character recognition system
- Salvi et.al. (2013) proposed algorithm based on average longest paths



Longest Path Problem: Application

Static timing analysis (STA)

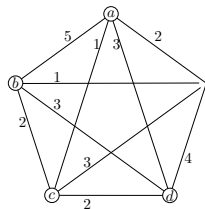
A simulation method of computing the expected timing of a digital circuit without requiring to simulate the full circuit. STA is performed only on these critical paths

▷ Widely used method in circuit design and embedded systems

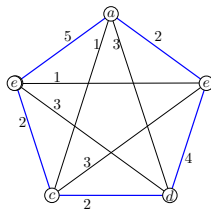
- Consider circuit components as graph nodes
- Connections between components are edges
- Find longest path between input and output component
- Longest path identifies critical paths in an IC or VLSI system

Traveling Salesman Problem

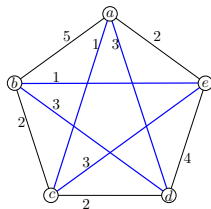
Given a complete graph G on n vertices with edge weights, a TSP tour is a Hamiltonian cycle in G



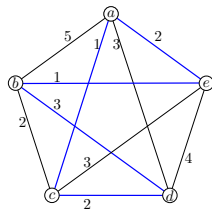
K_5 with edge weights



A TSP tour of length 15



A TSP tour of length 11



A TSP tour of length 9

Traveling Salesman Problem $\text{TSP}(G, w)$: Is there a TSP tour of weight w ?

- **Transportation:** A salesman wants to visit all cities with minimum cost
- **Optimize the tool path** for manufacturing equipment

