

Intractable Problems

- Clique
- Independent Set
- Vertex Cover
- Set Cover
- Set Packing
- Satisfiability Problem
- Hamiltonian Cycle and Path
- Traveling Salesman Problem
- Graph Coloring
- Circuit Satisfiability
- Knapsack
- Subset Sum
- Prime and Factor
- Partition

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Efficiently Solvable Problems

So far we dealt with problems like sort n numbers, find connected components, find shortest $s \rightarrow t$ path, find MST, find the best alignment, find matching

- We devised **efficient** algorithms for them

Efficient in the sense that the search space generally is exponential

▷ **Brute force algorithm would take exponential time**

- Only one ordering out of $n!$ permutation is sorted
 - Out of the possible n^{n-2} spanning trees (for K_n) only one is a MST
 - There could be exponentially many paths from s to t
 - Exponentially many alignments between two sequences
- Used greedy algorithms, dynamic programming to avoid exponential time
 - ▷ **Divide and Conquer typically is used to reduce already polynomial time**

Efficiently Solvable Problem

\exists an $O(n^k)$ worst case time algorithm for instances of size n , constant k

- Does not mean that n^{70} is OK, or no difference between n^2 and n^3
- We try to improve the polynomial's degree for polynomial time algorithms
 - Such as divide and conquer or design better data structures

Hard (Intractable) Problems

Efficiently Solvable Problem

\exists an $O(n^k)$ worst case time algorithm for instances of size n , constant k

- Now we study negative results
- Characterize problems for which we don't have good news
- **Cannot say they are not efficiently solvable (just don't know yet)**
- We might need to focus on approximation or special cases

Hard (Intractable) Problem

- No known $O(n^k)$ algorithm
- Exponential time is sufficient $O(n^n)$, $O(n!)$, $O(k^n)$

We establish that **these “hard problems”** are in some sense are equivalent

Hard Problems: Genres of Problems

We discuss six basic genres of hard problems and paradigmatic examples

- **Packing problems:** SET-PACKING, INDEPENDENT-SET
- **Covering problems:** SET-COVER, VERTEX-COVER
- **Constraint satisfaction problems:** SAT, 3-SAT
- **Sequencing problems:** HAMILTONIAN-CYCLE, TSP
- **Numerical problems:** SUBSET-SUM, KNAPSACK
- **Partitioning problems:** 3D-MATCHING, 3-COLORING
- **Number Theory problems:** FACTOR, DISCRETE-LOG