

Basic Graph Algorithms

- Exploring Graphs
- Depth First Search
- DFS Forest - Start and Finish Time
- DAG, Topological Sorting
- Strongly Connected Components
- Breadth First Search
- Bipartite Graphs

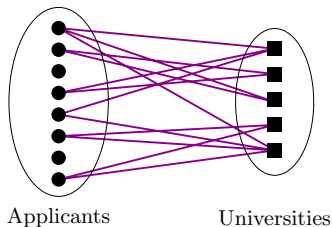
IMDAD ULLAH KHAN

Bipartite Graph

Bipartite Graph: $G = (V, E)$ is bipartite if V can be partitioned into two (disjoint and non-empty) subsets L and R , such that every edge $e \in E$ has one endpoint in L and other in R

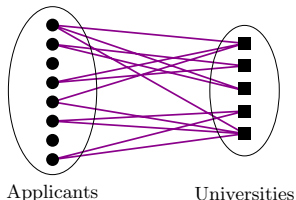
- for every edge $(u, v) \in E$, $u \in L$ and $v \in R$ or vice versa
- equivalently, no edge has both end points in L or in R

Often denoted by $G = (L, R, E)$



Bipartite Graph

- Bipartite graphs are bichromatic:
- $\chi(\mathbf{G}) = 2$
- Can color its vertices with 2 colors such that no two end points of an edge get the same color

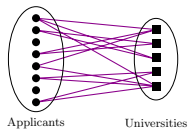


Bipartite Graph

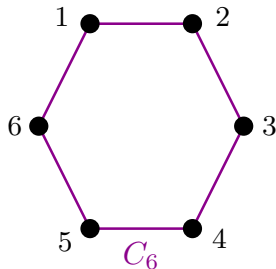
Bipartite graphs are

bichromatic:

$$\chi(G) = 2$$



Is C_6 bipartite?

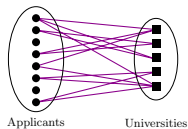


Bipartite Graph

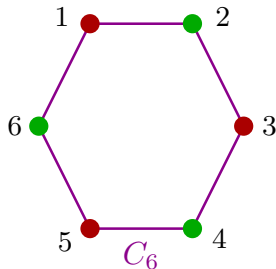
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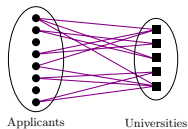


Bipartite Graph

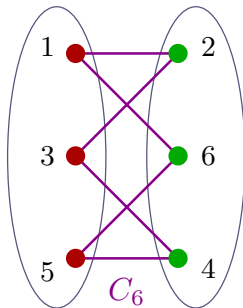
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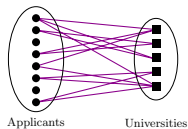


Bipartite Graph

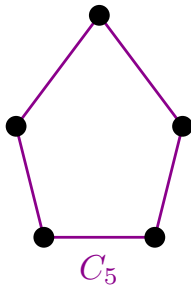
Bipartite graphs are

bichromatic:

$$\chi(G) = 2$$



Is C_5 bipartite?

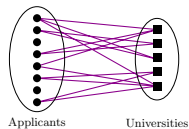


Bipartite Graph

Bipartite graphs are

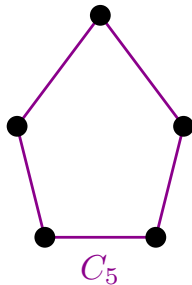
bichromatic:

$$\chi(G) = 2$$



Is C_5 bipartite?

For which n ,
 C_n is bipartite?



Bipartite Graph

For which n , C_n is bipartite?

C_n is bipartite, when n is even

C_n is not bipartite, when n is odd

Theorem

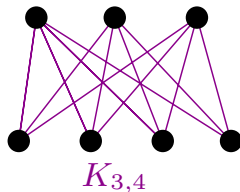
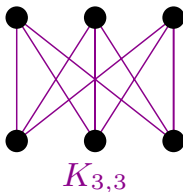
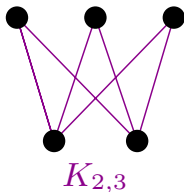
A graph is bipartite if and only if it contains no odd-length cycles

Complete Bipartite Graph

Bipartite Graph: $G = (V, E)$ is bipartite if V can be partitioned into two (disjoint and non-empty) subsets L and R , such that every edge $e \in E$ has one endpoint in L and other in R

It is a **complete bipartite graph** if all possible edges are present

Denoted by $K_{m,n}$

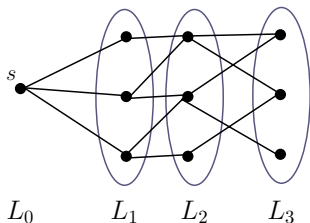


Bipartite-ness via BFS

A graph is bipartite if and only if it has no odd-length cycles

Graph bipartite-ness can be tested using BFS

- **Case 1** : There are no edges between the nodes of same layer

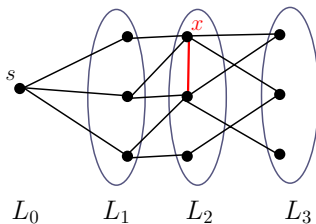


Bipartite-ness via BFS

A graph is bipartite if and only if it has no odd-length cycles

Graph bipartite-ness can be tested using BFS

- **Case 2** : There are some edges between the nodes of same layer



- $d(s, x) = i$ and $d(x, s) = i$
- cycle of length $2i + 1$ (graph not bipartite)