Polynomial Time Reduction

- Polynomial Time Reduction Definition
- Reduction by Equivalence
- Reduction from Special Cases to General Case
- Reduction by Encoding with Gadgets
- Transitivity of Reductions
- Decision, Search and Optimization Problem
- Self-Reducibility

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Polynomial Time Reduction

Problem A is polynomial time reducible to Problem B, $A \leq_p B$

If any instance of problem A can be solved using a polynomial amount of computation plus a polynomial number of calls to a solution of problem B

If any algorithm for problem B can be used [called (once or more) with *'clever'* legal inputs] to solve any instance of problem A

Subroutine for B takes an instance y of B and returns the solution $B(y)$

Algorithm for A transforms an instance x of A to an instance y of B. Then transforms $B(y)$ to $A(x)$

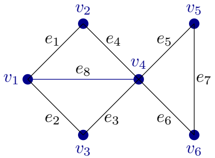
Reduction from special case to general case

VERTEX-COVER(G, k) $\leq_p$ SET-COVER$(U, \mathcal{S}, k')$

Let $\mathcal{A}$ be an algorithm solving SET-COVER$(U, \mathcal{S}, k')$

Let $[G, k]$ be an instance of the VERTEX-COVER problem

1. Make $U = E$, $\quad k' = k$
2. $\mathcal{S} = \{S_1, \ldots, S_n\}$, where $S_i = \{e \in E \mid e \text{ is incident on } v_i\}$



$U = \{e_1, e_2, e_2, e_4, e_5, e_6, e_7\}$

$S_1 = \{e_1, e_2, e_8\}$
$S_3 = \{e_2, e_3\}$
$S_2 = \{e_1, e_4\}$
$S_4 = \{e_3, e_4, e_5, e_6, e_8\}$
$S_5 = \{e_5, e_7\}$
$S_6 = \{e_6, e_7\}$

Theorem: $[U, \mathcal{S}]$ has a set cover of size k iff G has a vertex cover of size k

3. If $\mathcal{A}(U, \mathcal{S}, k') =$ **Yes**, then output **Yes**, else output **No**

Reduction from special Case to General Case

We get the following reduction very similarly

$$\textsc{independent-set}(G, k) \ \leq_p \ \textsc{set-packing}(U, \mathcal{S}, k')$$

The following reductions are even more straight forward. They follow from respective definitions of the problems

$$3\text{-}\textsc{sat}(f) \ \leq_p \ \textsc{sat}(f')$$

$$\textsc{subset-sum}(U, w, C) \ \leq_p \ \textsc{knapsack}(U, w, v, C')$$

Please complete their details. Explicitly and formally writing them will help understand the important notion of reduction