Problem Set: Asymptotic Analysis

Problem 1. Show that 2^{n+1} is $O(2^n)$

Problem 2. In each case below, demonstrate whether the given expression is true or false.

a) $5n^3 \in O(n^3)$

b) $100n^2 \in O(n^4)$

c) $\log n^2 \in O(\log n)$

d) $(n^2 + 7n - 10)^3 \in O(n^6)$

e) $\sqrt{n} \in O((\log n)^3)$

Problem 3. In each row of the following table, write whether $f = O(g)$, or $f = \Omega(g)$, or both i.e. $f = \Theta(g)$

$f(n)$	$g(n)$	Your Answer
100	17	
10^{200}	$n - 10^{200}$	
$n^2 \log 300$	$n \log n$	
n^{100}	2^n	
$n \log n$	$n - 100$	
$\sqrt{n}$	$\log n$	
$n^{1.01}$	$n + 100$	
$2 \log n$	$\log(n^2)$	
$\log(n^2)$	$(\log n)^2$	

Problem 4. Order the following functions of n from the smallest to largest complexity. If two have the same complexity put them in one line next to each other.

$$n \log n, \quad (n-5)(n), \quad 10 \log 300, \quad n + 7\log(n^2),$$

$$2^n, \quad 10n^2 + 15n, \quad 17, \quad 15n^3 + n \log n, \quad \frac{3n^8}{n^6}$$

$$10^{200}, \quad n^2 \log 300, \quad 2\log_{10} 10^{200}, \quad n^2 + 7\log(n^2),$$

$$3^{n^2}, \quad n^2 + \sqrt{n}, \quad n^2 \log n, \quad n^4 + n^2 + n^2 \log n, \quad n^4$$

Problem 5. **[Complexity Classes]** Order the following functions of n from the smallest to largest complexity and also identify the complexity class for each. If two have the same complexity put them in one line next to each other. Briefly explain next to each line why these functions are in the same complexity class.

$$10^{200}, \quad n^2 \log 300, \quad 2\log_{10} 10^{200}, \quad n^2 + 7\log(n^2),$$

$$3^{n^2}, \quad n^2 + \sqrt{n}, \quad n^2 \log n, \quad n^4 + n^2 + n^2 \log n, \quad n^4$$

Problem 6. Let $f(n)$ and $g(n)$ by asymptotically positive functions. Prove or disprove each of the following conjectures.

1. $f(n) = O(g(n)) \implies g(n) = O(f(n))$.
2. $f(n) + g(n) = \Theta(\min(f(n), g(n)))$.
3. $f(n) = O(g(n)) \implies \lg(f(n)) = O(\lg(g(n)))$, where $\lg(g(n)) \geq 1$ and $f(n) \geq 1$ for all sufficiently large n.
4. $f(n) = O(g(n)) \implies 2^{f(n)} = O(2^{g(n)})$.
5. $f(n) = O((f(n))^2)$.
6. $f(n) = O(g(n)) \implies g(n) = \Omega(f(n))$.
7. $f(n) = \Theta(f(n/2))$.
8. $f(n) + o(f(n)) = \Theta(f(n))$.

Problem 7. Prove or disprove: it is asymptotically faster to square an n-bit integer than to multiply two n-bit integers.