Trees and other Special Classes of Graphs

- Special Classes of Graphs
  - Complete Graphs, Path, Cycle, Star, Wheel, *n*-Cubes
- Bipartite Graphs
- Trees
  - Characterization of Trees
  - Minimum Spanning Tree
  - Rooted Trees

IMDAD ULLAH KHAN

Bipartite Graphs

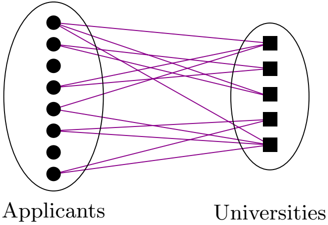
A graph $G = (V, E)$ is **bipartite** if

V can be partitioned into two disjoint non-empty subsets L and R

such that no edge in G connects two vertices in L or two vertices in R

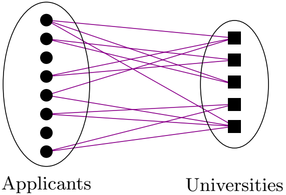
▷ i.e. all edges are between the parts L and R

Often denoted by $G = (L, R, E)$



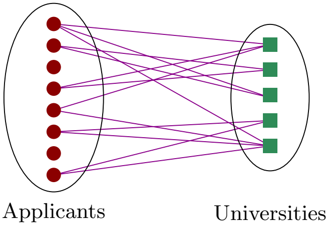
Bipartite Graphs

In many applications the problem is modeled with bipartite graphs



- Actors & Movies
- Artists & Albums
- Authors & Papers
- Users & Online groups
- Words & Documents
- Users & Checkins locations
- Metabolites & Reactions

Bipartite Graphs



Bipartite graphs are bichromatic

▷ Its vertices can be colored with 2 colors

$$\chi(G) = 2$$

Bipartite Graphs

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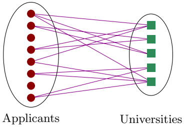
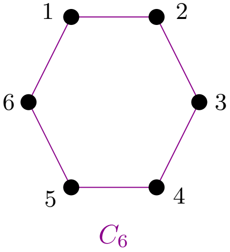
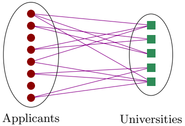


Is C_6 bipartite?



Bipartite Graphs



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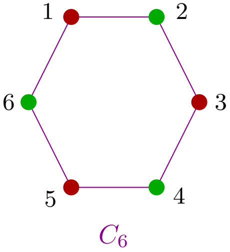
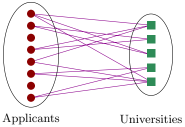


Bipartite Graphs



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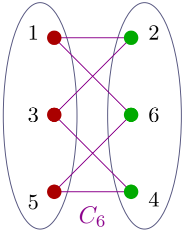
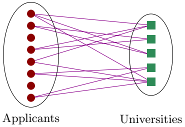
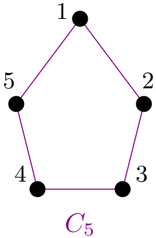


Bipartite Graphs

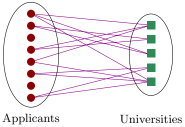


Bipartite graphs are bichromatic: $\chi(G) = 2$

Is C_5 bipartite?

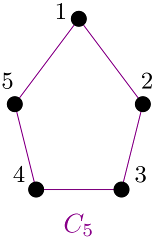


Bipartite Graphs



Bipartite graphs are bichromatic: $\chi(G) = 2$

For which n, C_n is bipartite?



Bipartite Graphs

ICP 15-07 For which n, C_n is bipartite ?

C_n is bipartite, when n is even

C_n is not bipartite, when n is odd

Theorem

A graph is bipartite if and only if it contains no odd-length cycles

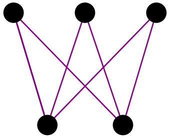
Complete Bipartite Graphs

A graph $G = (V, E)$ is **bipartite** if

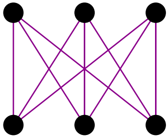
- V can be partitioned into two disjoint non-empty subsets L and R
- such that no edge in G connects two vertices in L or two vertices in R
- i.e. all edges are between the parts L and R

It is a complete bipartite graph if all possible edges are present

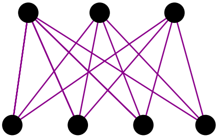
Denoted by $K_{m,n}$



$K_{2,3}$



$K_{3,3}$



$K_{3,4}$

ICP 15-08 How many edges are there in $K_{m,n}$? mn