# Graphs

- Graphs are everywhere
- Types and Terminology: Handshaking lemma
- Representation, Complement, Transpose, Subgraph
- Walks, Paths and Cycles
- (Strongly) Connected and $k$-Connected graphs
- Applications: BFS, DFS, Eulerian graphs
- Advanced Applications: Optimization & Massive Graph Analysis

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# Ferryman transportation puzzle

# Ferryman transportation puzzle

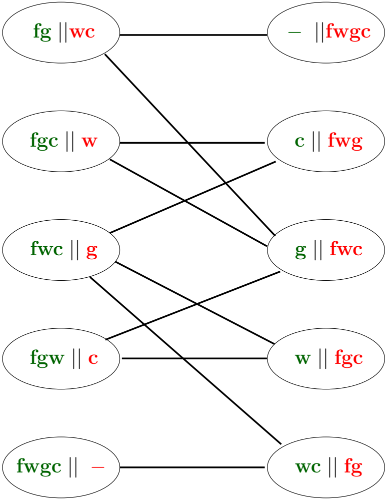
Ferryman wants to transport all 3 objects to the other side

- Boat can carry one object with ferryman
- Wolf cannot be alone with goat
- Goat cannot be alone with cabbage

# Ferryman transportation puzzle

Represent state of objects as vertex *fw* || *gc*

An edge implies possible transition in one trip



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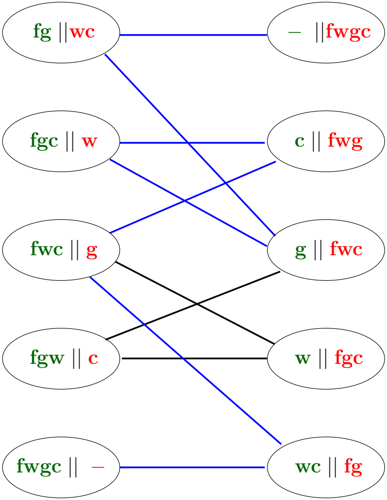
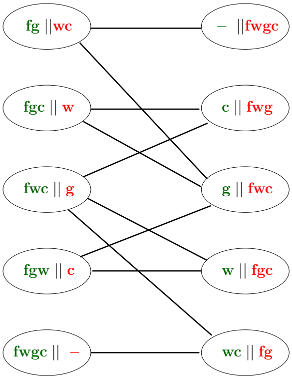


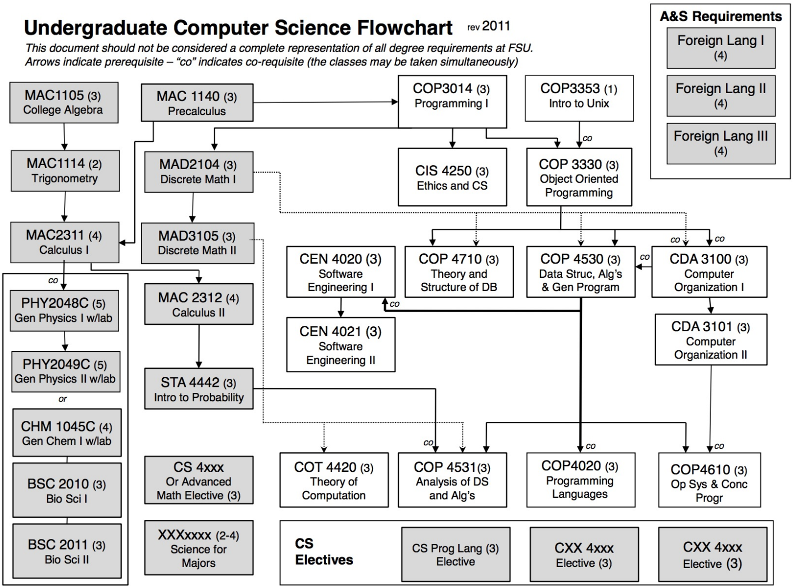
# Ferryman transportation puzzle

Find a path from one vertex (source) to another (target)

Breadth First Search (BFS) Algorithm accomplishes this



# Directed Acyclic Graph: DAG



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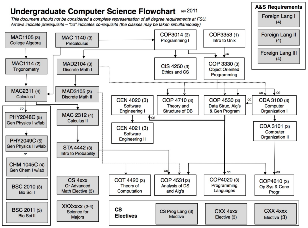



Make a graph: vertices represent courses

Directed edges represent pre-requisites

Can there by cycle(s) in this graph?

**Directed Acyclic Graph: DAG**

What could be a feasible order for a student to take these courses?

Topological sort of $V(G)$: An ordering of vertices with all edges directed from left to right

Depth First Search (DFS)

# Graph Connectivity

## $k$-Connected Graph

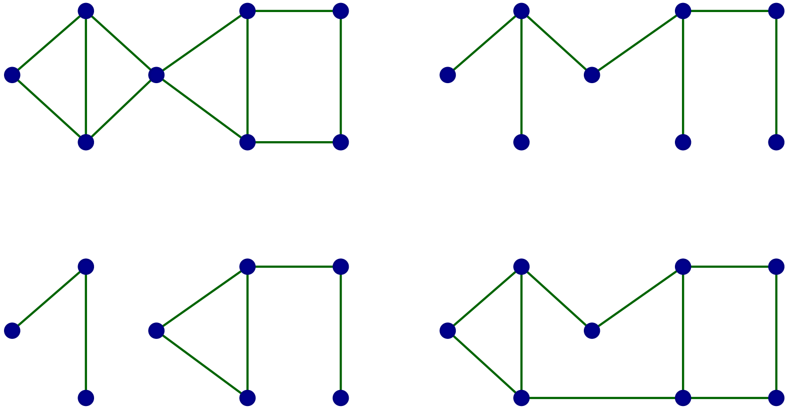
A connected graph is $k$-connected if it remains connected after removing $k-1$ vertices

## Cut Vertex

A vertex whose removal makes the graph disconnected (or increase the number of connected components)

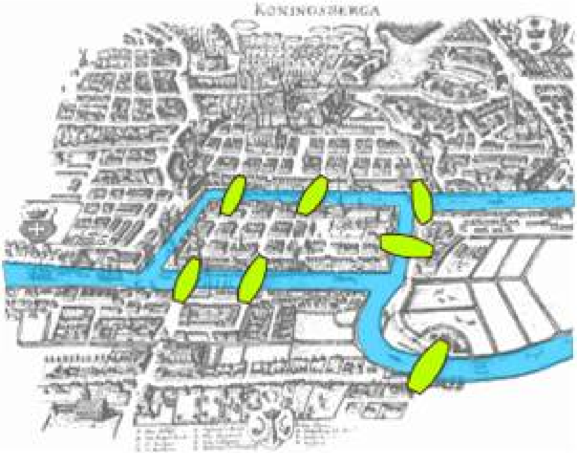
# Graph Connectivity

Which one is a good design for a network ?



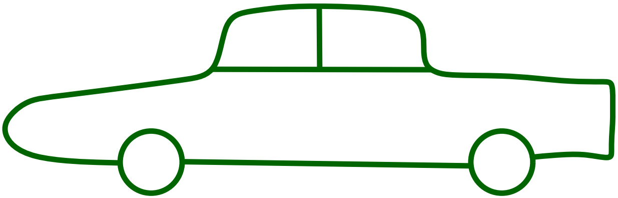
# Eulerian Graphs

Tour this city traveling each bridge exactly once



# Eulerian Graphs

Draw this picture without lifting pencil or retracing



# Eulerian Graphs

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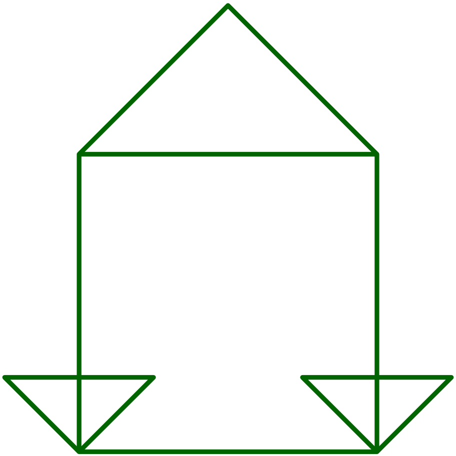
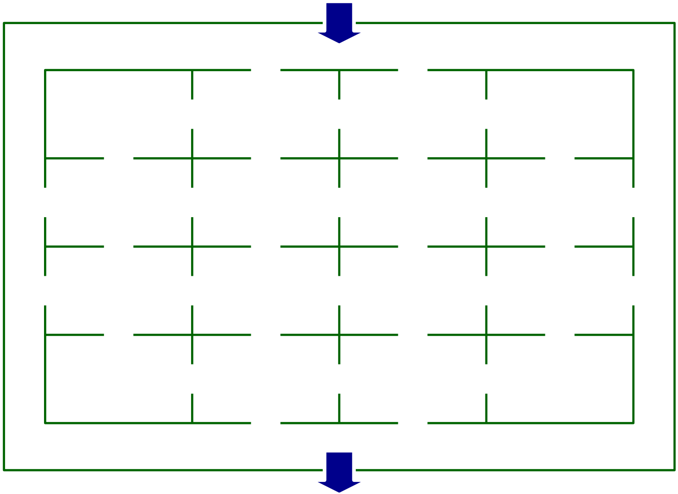


# Eulerian Graphs

Tour the building passing each door exactly once



# Eulerian Graphs

## Euler Circuit

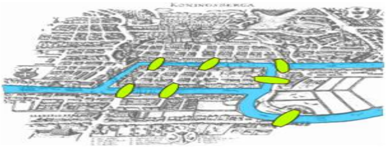
A closed walk in $G$ containing every edge of $G$ exactly once

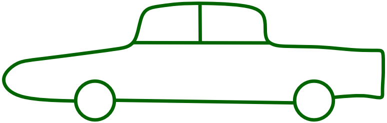
## Euler Path

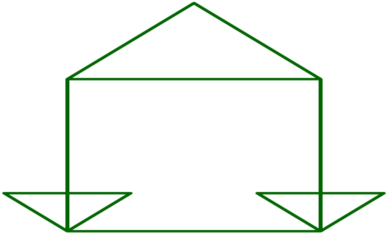
A walk in $G$ containing every edge of $G$ exactly once

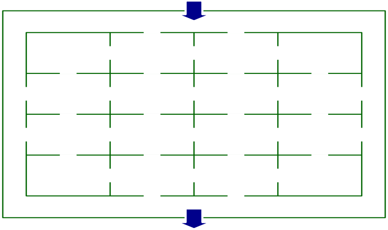
# Eulerian Graphs

Which graphs has Euler Path/Circuit?









# Eulerian Graphs

**Theorem**

*$G$ contains an Euler circuit if and only if every vertex has even degree*

**Theorem**

*$G$ contains an Euler path if and only if it has exactly two vertices of odd degree*

Proofs of these theorems are in your textbook