Graphs

- Graphs are everywhere
- Types and Terminology: Handshaking lemma
- Representation, Complement, Transpose, Subgraph
- Walks, Paths and Cycles
- (Strongly) Connected and k-Connected graphs
- Applications: BFS, DFS, Eulerian graphs
- Advanced Applications: Optimization & Massive Graph Analysis

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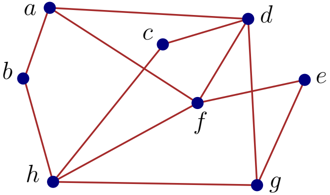
Graph Connectivity

In an undirected graph a pair of vertices u **and** v **are connected** if there is a path between u and v

Do not mix-up this notion with that of u and v being adjacent

If u and v are adjacent, then u is connected to v

▷ The converse is not necessarily true



ICP 14-38 Are c and d adjacent?

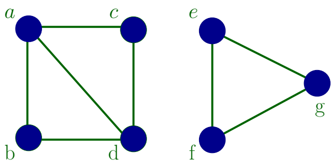
ICP 14-39 Are c and d connected?

ICP 14-40 Are a and g adjacent?

ICP 14-41 Are a and g connected?

Graph Connectivity

In an undirected graph a pair of vertices u **and** v **are connected** if there is a path between u and v



ICP 14-42 Are c and d adjacent?

ICP 14-43 Are c and d connected?

ICP 14-44 Are b and c adjacent?

ICP 14-45 Are b and c connected?

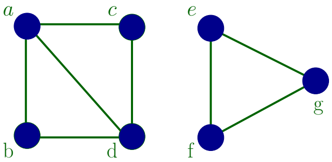
ICP 14-46 Are a and e adjacent?

ICP 14-47 Are a and e connected?

ICP 14-48 Are f and g connected?

Graph Connectivity

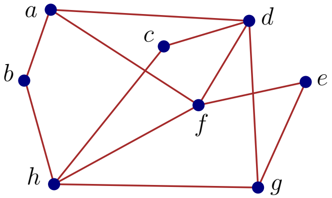
An **undirected graph is connected** if all pairs of distinct vertices are connected



ICP 14-49 Are b and c connected?

ICP 14-50 Is every pair connected?

ICP 14-51 Is the graph connected?

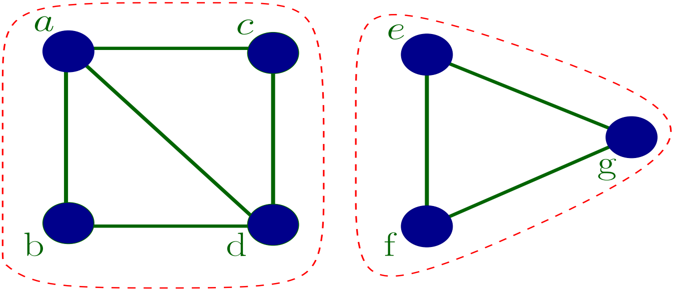


ICP 14-52 Are a and g connected?

ICP 14-53 Is every pair connected?

ICP 14-54 Is the graph connected?

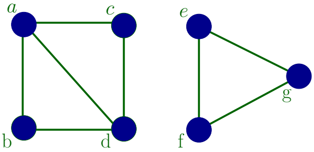
A **connected component** of G is a maximal connected subgraph (every possible connected vertex is included)



Connected components of G

Graph Connectivity

A **connected component** of G is a maximal connected subgraph (every possible connected vertex is included)



ICP 14-55 Is the graph connected?

ICP 14-56 Is the subgraph induced by $\{e, f, g\}$ connected?

ICP 14-57 Is the subgraph induced by $\{e, f, g\}$ a connected component?

ICP 14-58 Is the subgraph induced by $\{a, b, c\}$ connected?

ICP 14-59 Is the subgraph induced by $\{a, b, c\}$ a connected component?

Undirected Graph Connectivity

In an undirected graph **u and v are connected** if there is a path between u and v

An **undirected graph is connected** if all pairs of distinct vertices are connected

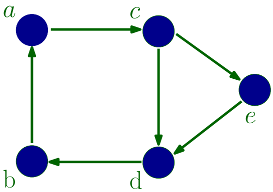
- i.e. if there is a path between every pair of distinct vertices

A **connected component** of G is a maximal connected subgraph (every possible connected vertex is included)

$\triangleright$ A subset of vertices in which all pairs are connected and no other vertex can be added

Digraph Connectivity

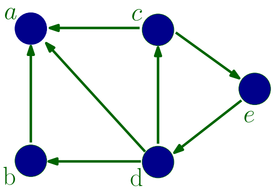
In a digraph u and v are **strongly connected**, if there is a path from u to v AND a path from v to u



ICP 14-60 Is there a path from a to e?

ICP 14-61 Is there a path from e to a?

ICP 14-63 Are a and e strongly connected?



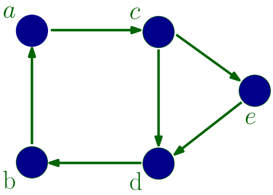
ICP 14-64 Is there a path from a to e?

ICP 14-65 Is there a path from e to a?

ICP 14-66 Are a and e strongly connected?

Digraph Connectivity

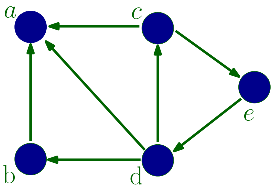
A **digraph is strongly connected**, if every pair of distinct vertices are strongly connected



ICP 14-70 Are c and d strongly connected?

ICP 14-71 Are all pairs strongly connected?

ICP 14-72 Is the graph strongly connected?



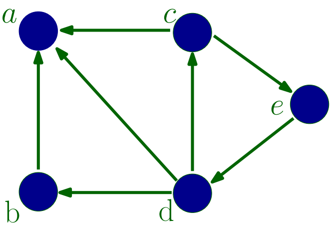
ICP 14-73 Are c and d strongly connected?

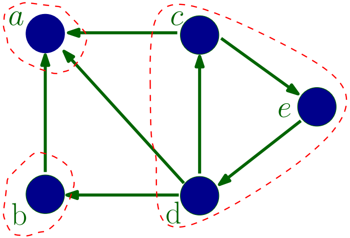
ICP 14-74 Are all pairs strongly connected?

ICP 14-75 Is the graph strongly connected?

Digraph Connectivity

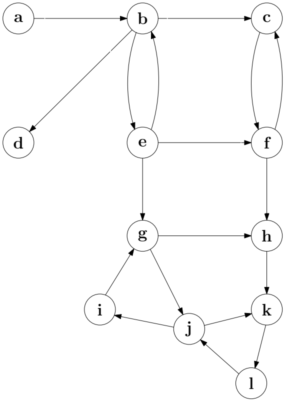
A **strongly connected component** in a digraph is a maximal strongly connected subgraph (every possible strongly connected vertex is included)





Digraph Connectivity

A **strongly connected component** in a digraph is a maximal strongly connected subgraph (every possible strongly connected vertex is included)



ICP 14-76 Are a and b strongly connected?

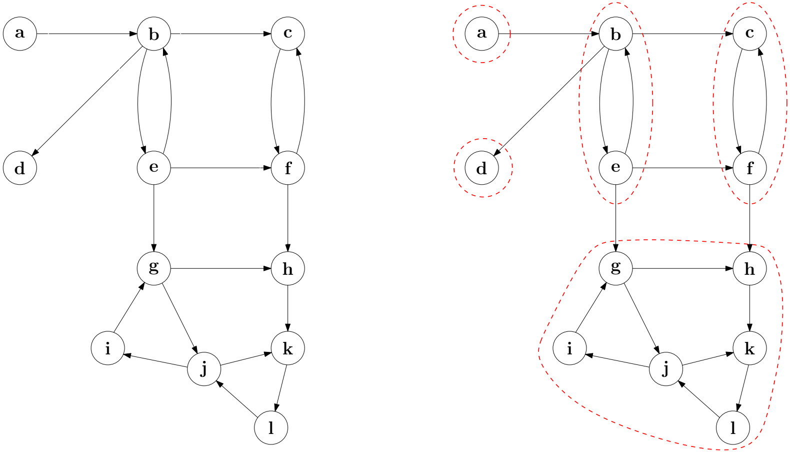
ICP 14-77 Are b and e strongly connected?

ICP 14-78 Is the subgraph induced by $\{j, k, l\}$ strongly connected?

ICP 14-79 Is the subgraph induced by $\{j, k, l\}$ a strongly connected component?

Digraph Connectivity

A **strongly connected component** in a digraph is a maximal strongly connected subgraph (every possible strongly connected vertex is included)



Directed Graph Connectivity

In a digraph u and v are **strongly connected**, if there is a path from u to v AND a path from v to u

A **digraph is strongly connected**, if every pair of distinct vertices are strongly connected

A **strongly connected component** in a digraph is a maximal strongly connected subgraph (every possible strongly connected vertex is included)

Connectivity of Graph Complement

Theorem

The complement of a disconnected graph is connected

ICP 14-80 Give a formal proof of this theorem

This is a very important and straight forward result

▷ Simply follows from definition of vertices and graph connectivity

If you cannot prove this, you need to understand the above undirected graph connectivity definitions