Graphs

- Graphs are everywhere
- Types and Terminology: Handshaking lemma
- Representation, Complement, Transpose, Subgraph
- Walks, Paths and Cycles
- (Strongly) Connected and k-Connected graphs
- Applications: BFS, DFS, Eulerian graphs
- Advanced Applications: Optimization & Massive Graph Analysis

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Graph Connectivity

Walk

A walk in a digraph is a sequence of vertices

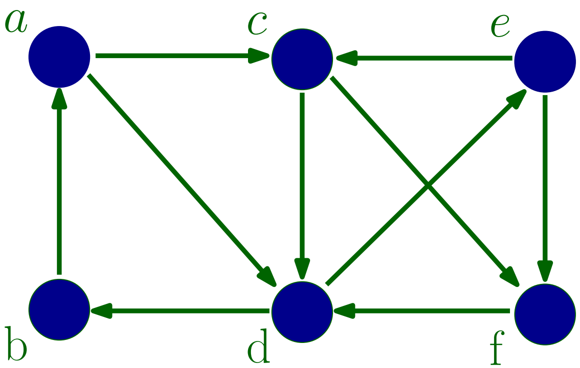
$$v_1, v_2, \ldots, v_k$$

such that $(v_i, v_{i+1}) \in E \quad$ for $\quad 1 \leq i \leq k-1$

Just follow successive edges

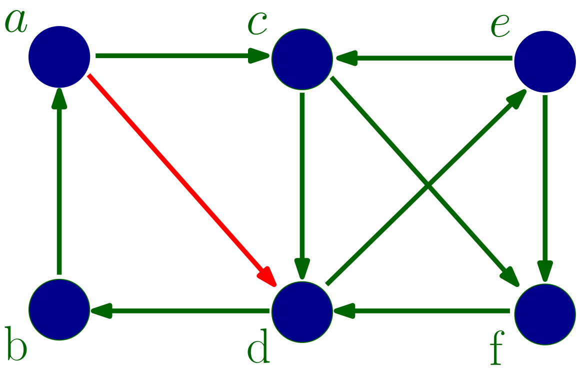
Walk

A Walk



Walk

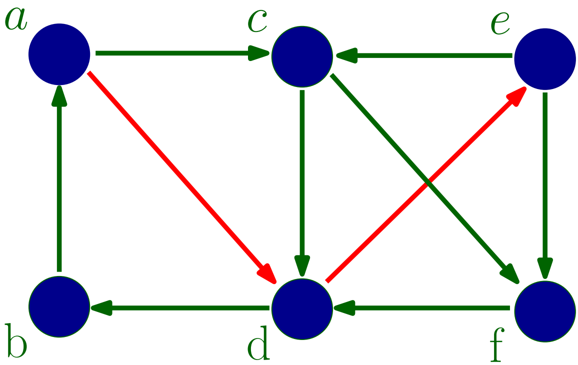
A Walk



a, d

Walk

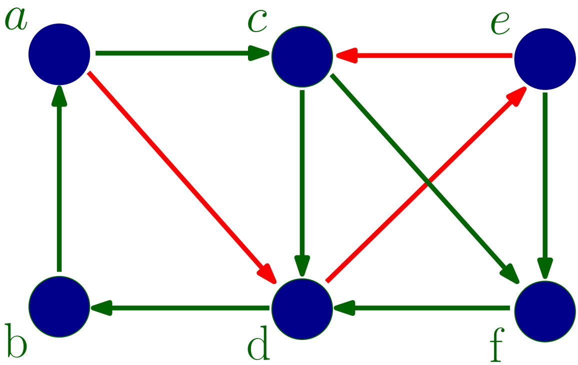
A Walk



a, d, e

Walk

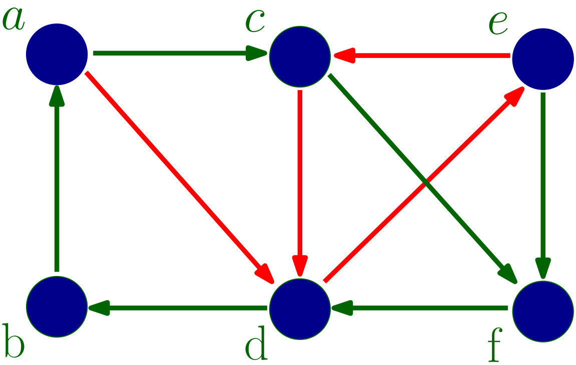
A Walk



a, d, e, c

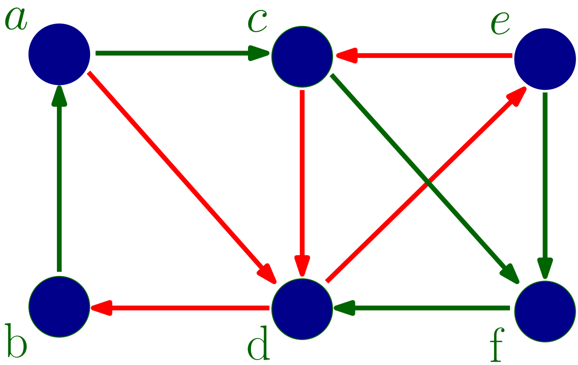
Walk

A Walk



a, d, e, c, d

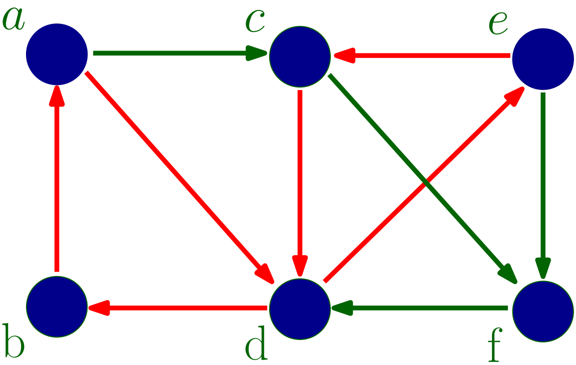
A Walk



a, d, e, c, d, b

Walk

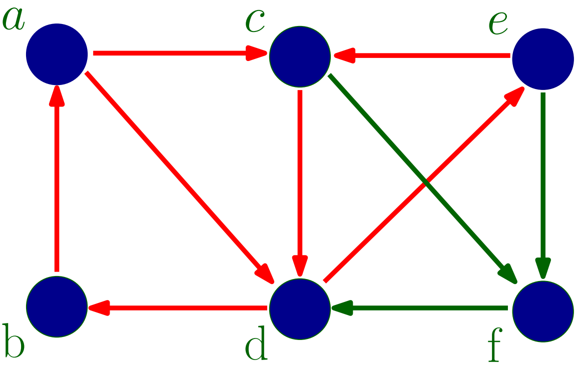
A Walk



a, d, e, c, d, b, a

Walk

A Walk



a, d, e, c, d, b, a, c

Length of a Walk

Walk

A walk in a digraph is a sequence of vertices

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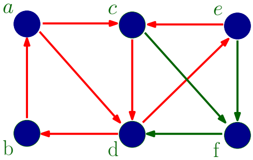
such that $(v_i, v_{i+1}) \in E \quad \text{for} \quad 1 \leq i \leq k-1$



a, d, e, c, d, b, a, c

Length of a walk is the number of edges in it

Path

Path

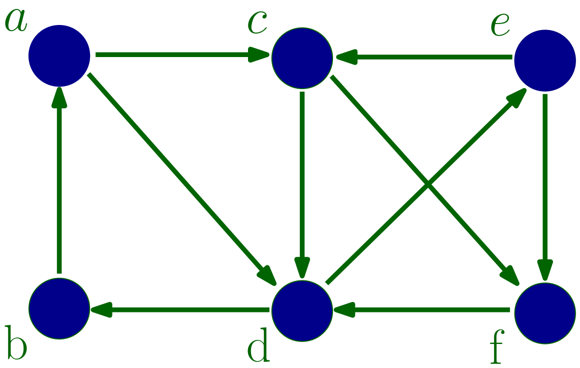
A path in a digraph is a sequence of vertices with no repetition

$$v_1, v_2, \ldots, v_k$$

such that $(v_i, v_{i+1}) \in E \quad \text{for} \quad 1 \leq i \leq k-1$

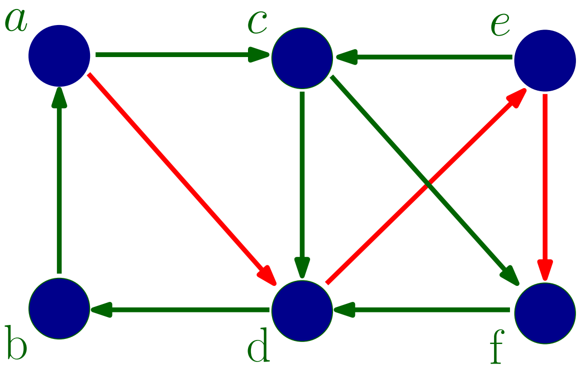
Path

A Path



Path

A Path



a, d, e, f

Length of a Walk

Path

A path in a digraph is a sequence of vertices with no repetition

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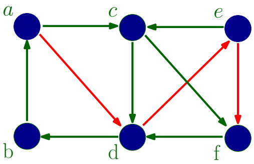


a, d, e, f

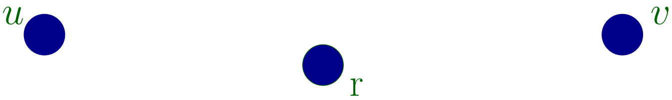
Length of a path is the number of edges in it

Shortest Walk

Theorem

The shortest walk between u and v is a path

Suppose it is not a path and some vertex *r* is repeated



Shortest Walk

Theorem

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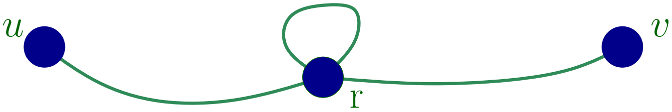


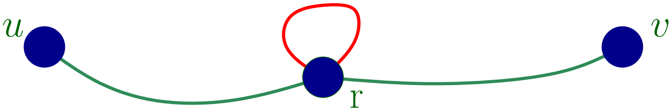
Shortest Walk

Theorem

The shortest walk between u and v is a path

Suppose it is not a path and some vertex r is repeated

The path without $r - - - - r$



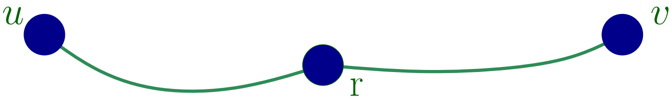
Shortest Walk

Theorem

The shortest walk between u and v is a path

Suppose it is not a path and some vertex r is repeated

The path without $r - - - - r$ is shorter!

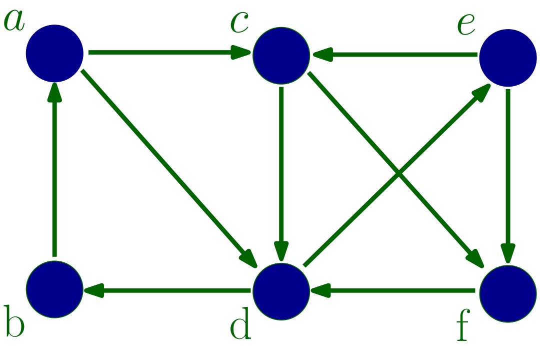


ICP 14-36 Give a formal proof of this theorem.

Closed Walk

Closed Walk

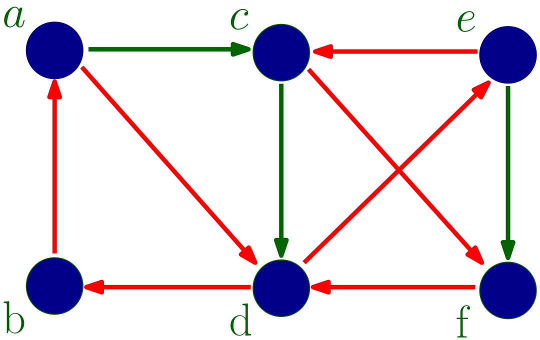
A walk that starts and ends at the same vertex



Closed Walk

Closed Walk

A walk that starts and ends at the same vertex

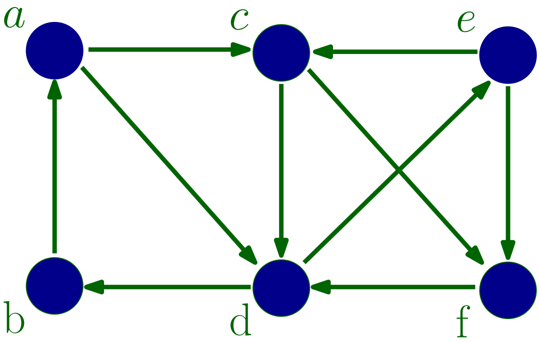


$$a, d, e, c, f, d, b, a$$

Cycle

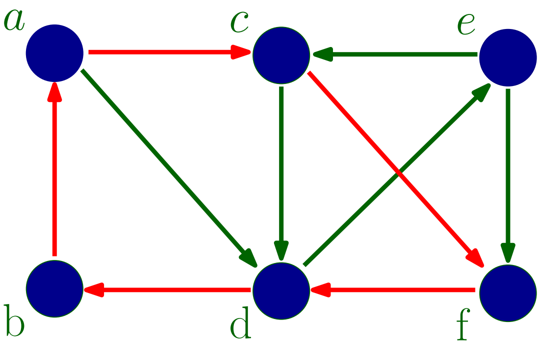
Cycle

A path that starts and ends at the same vertex



Cycle

A path that starts and ends at the same vertex



a, c, f, d, b, a

Cycle

Theorem

The shortest closed walk from u to u is a cycle

ICP 14-37 Give a formal proof of this theorem.

Proof is analogous to the proof of shortest walk being a path