

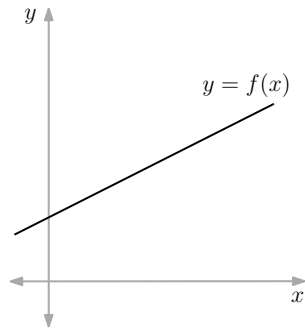
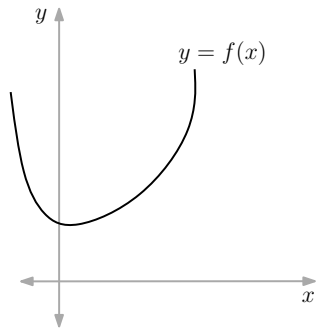
## Graphs

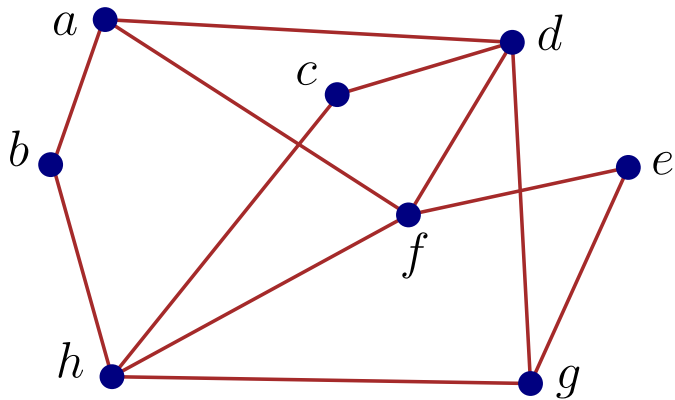
- Graphs are everywhere
- Types and Terminology: Handshaking lemma
- Representation, Complement, Transpose, Subgraph
- Walks, Paths and Cycles
- (Strongly) Connected and  $k$ -Connected graphs
- Applications: BFS, DFS, Eulerian graphs
- Advanced Applications: Optimization & Massive Graph Analysis

IMDAD ULLAH KHAN

# Continuous Math. Graphs

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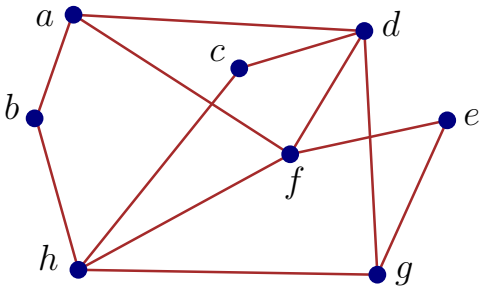
# Discrete Math. Graphs

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Intuitively:

Graphs are some vertices represented by ●

Vertices are connected by edges represented by **line segments**



# Graphs

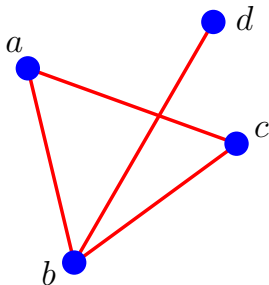
Formally: A graph is

- 1 a set of vertices  $V$
- 2 a set of edges  $E$ ; each edge is a 2-subset of  $V$

$$G = (V, E)$$

$$V = \{a, b, c, d\}$$

$$E = \{(a, b), (a, c), (b, c), (b, d)\}$$



Each edge is an unordered pair from  $V$ , we use the ordered pair notation

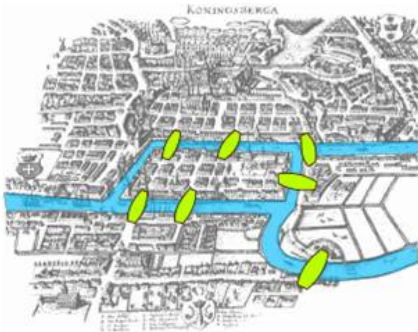
# Graph Analysis

Depending on the domain of graphs and applications the area is also called

Network Analysis, Link Analysis, Social Network Analysis

## Modeling, formulating and solving problems with graphs

Use tools from graph theory, linear algebra, algebraic graph theory, and algorithms for data analysis problems modeled with graphs



One of the earliest graph analysis:

Euler argued that there is no way to tour the city of Königsberg (now Kaliningrad) crossing each of the seven bridges exactly once.

# Graph Analysis

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Rather than individual data points or the global structure of the datasets

Graph Analysis focuses on pairwise interaction between objects

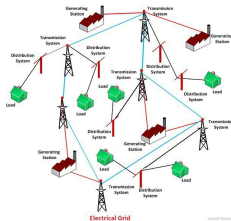
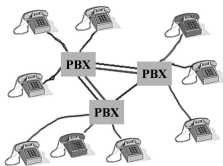
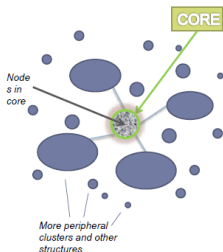
Allows to examine how pairwise interaction of entities in a network determine the behavior or function of

- an individual entity,
- groups of entities,
- or the whole system

# Graphs are everywhere: Six major classes of networks

## Technological Networks

- **The Internet** (Autonomous systems connected with BGP connections)
- **Telecom Network** (telephone devices connected with wires or wireless)
- **Power Grid** (generating stations/users and transmission line)

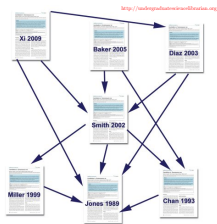
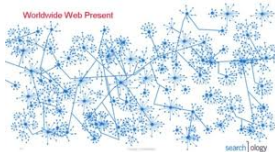
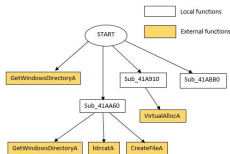




# Graphs are everywhere: Six major classes of networks

## Information Networks

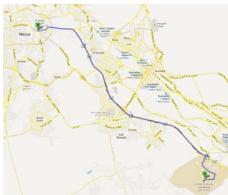
- **Software** (functions connected with function calls)
- **The Web Graph** (webpages and hyperlinks)
- **Citation Network** (Research papers and citations)



# Graphs are everywhere: Six major classes of networks

## Transportation Networks

- **Railway System** (train stations and railroad tracks)
- **Highway network** (Intersections and road segments)
- **Air Transportation** (Airports and non-stop flight)



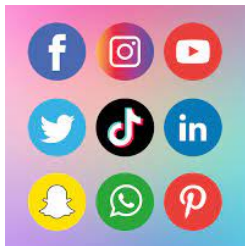
# Graphs are everywhere: Six major classes of networks

## Social Networks

- **Social network** (people and friendship/coworker relation)
- **Online Social Network** (people and friendship or following relation)



Social Network



Online Social Network

# Graphs are everywhere: Six major classes of networks

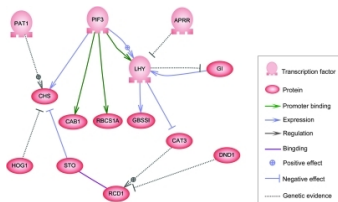
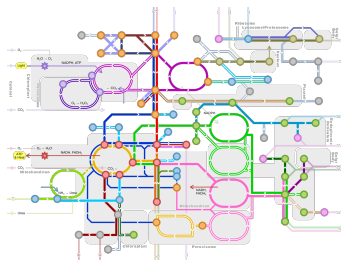
## Biological Networks

Represent interactions between biological units

ecological, evolutionary, physiological, metabolic, gene regulatory network

Most genes and proteins play a role through interactions with other proteins, genes, and biomolecules

Analyzed to understand the origin and function of cellular components, treatments for diseases, determine comorbidities and risk factors



# Graphs are everywhere: Six major classes of networks





## Economic Networks

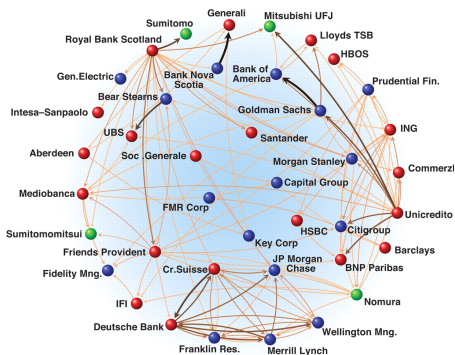
Business, companies, governments interacting via credit and investment, trade relations, supply chain

REVIEW article

Front. Appl. Math. Stat., 28 August 2018 | <https://doi.org/10.3389/fams.2018.00037>

### Understanding the World Economy in Terms of Networks: A Survey of Data-Based Network Science Approaches on Economic Networks

 Frank Emmert-Streib<sup>1,2</sup>,  Shailesh Tripathi<sup>3</sup>,  Olli Yli-Harja<sup>1,4</sup> and  Matthias Dehmer<sup>1,5</sup>



F. Schweitzer et al. (2009) Economic Networks: The New Challenge

## Graphs are everywhere

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Graph	Vertices	Edges	Flow
Communications	Telephones exchanges, computers, satellites	Cables, fiber optics, microwave relays	Voice, video, packets
Circuits	Gates, registers, processors	Wires	Current
Mechanical	Joints	Rods, beams, springs	Heat, energy
Hydraulic	Reservoirs, pumping stations, lakes	Pipelines	Fluid, oil
Financial	Stocks, currency	Transactions	Money
Transportation	Airports, rail yards, street intersections	Highways, railbeds, airway routes	Freight, vehicles, passengers