Binomial Theorem and Pascal Triangle

- The Binomial Theorem
- The Pascal Triangle
- Patterns in the Pascal Triangle (mod n)

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The Pascal Triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

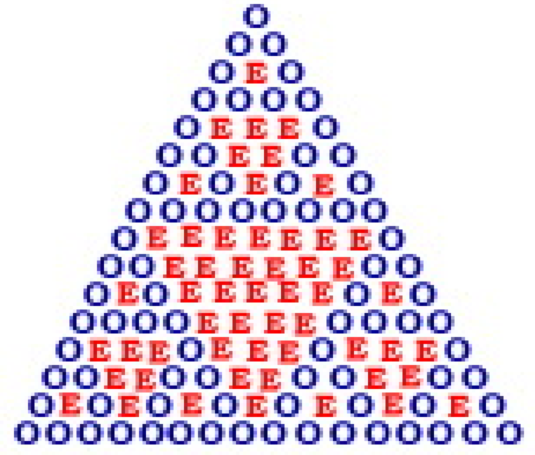
1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

$\binom{8}{0}$ $\binom{8}{1}$ $\binom{8}{2}$ $\binom{8}{3}$ $\binom{8}{4}$ $\binom{8}{5}$ $\binom{8}{6}$ $\binom{8}{7}$ $\binom{8}{8}$

The Pascal Triangle



1

[1]Image source: math.hmc.edu

Patterns in The Pascal Triangle

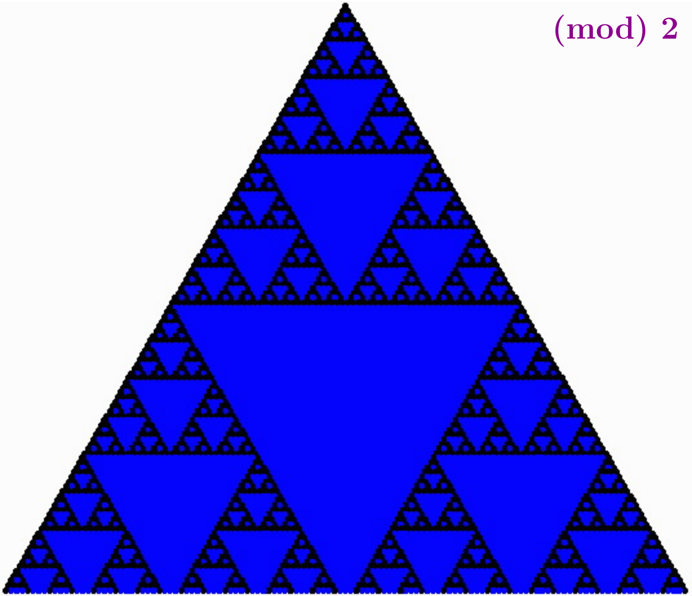
Observe the numbers in Pascal's Triangle mod n

Images credit:

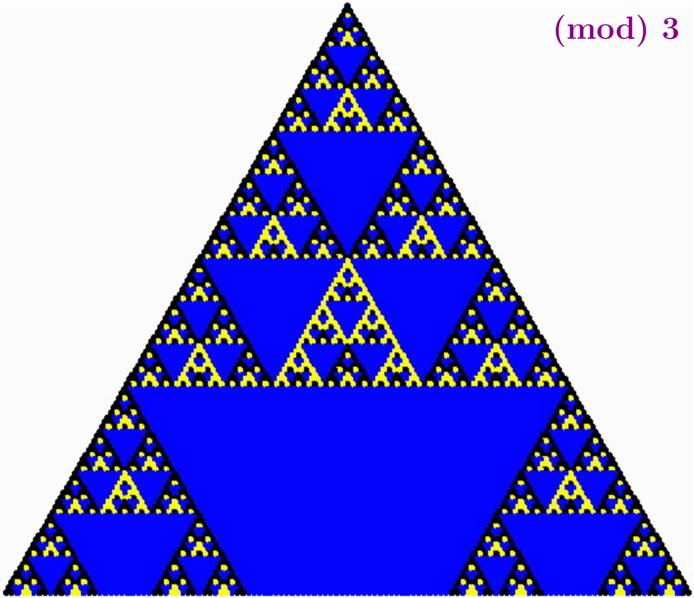
The Mathematical Association of America, Mathematical Sciences Digital Library.

Authors: Kathaleen Shannon and Michael Bardzell.

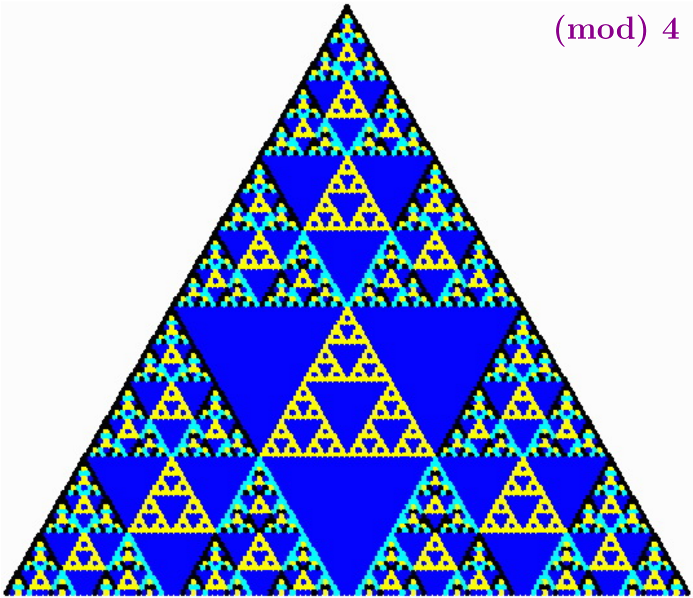
Patterns in The Pascal Triangle



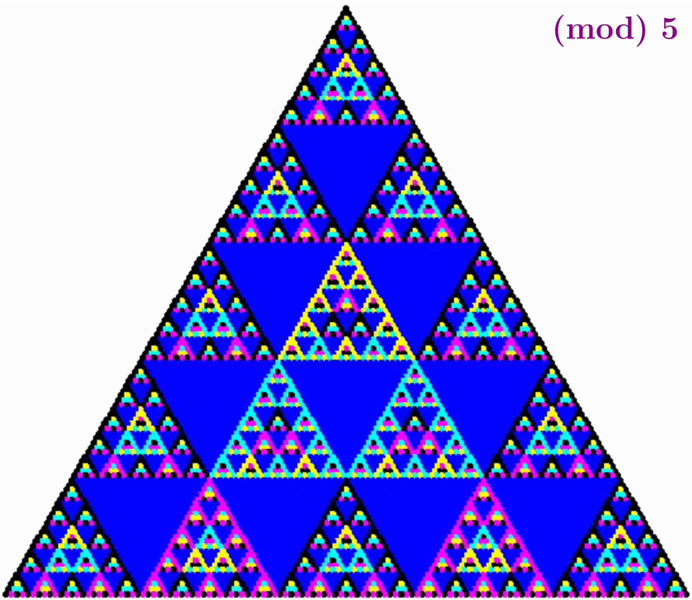
Patterns in The Pascal Triangle



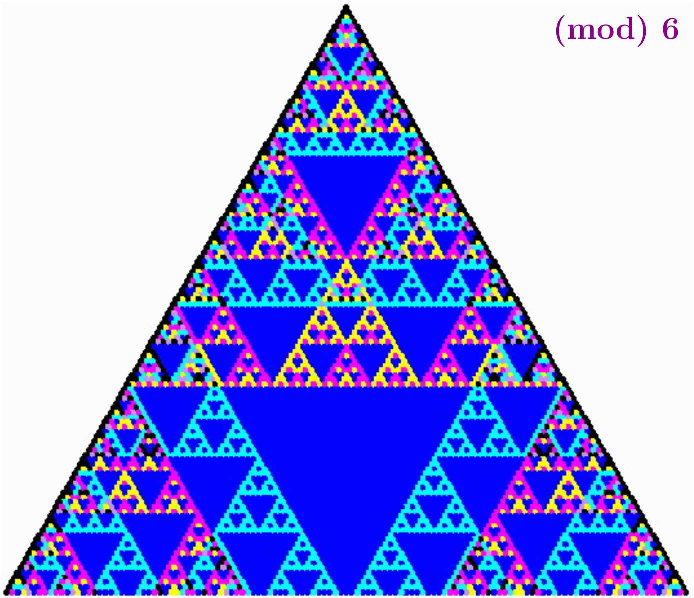
Patterns in The Pascal Triangle



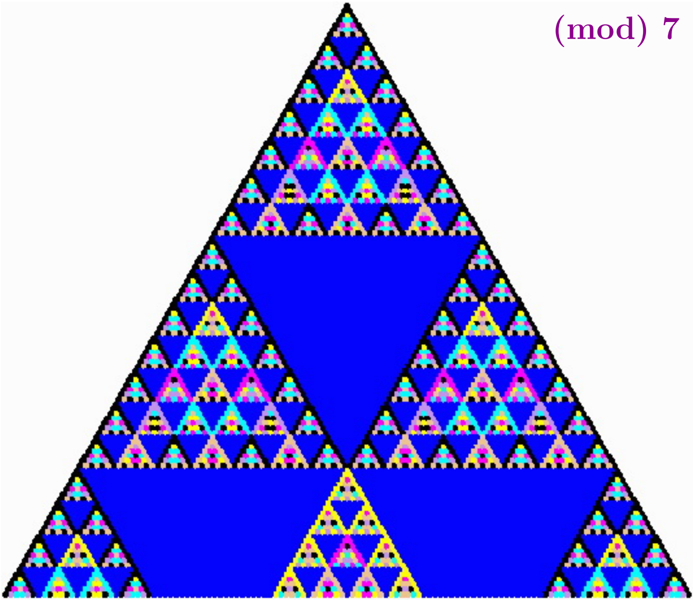
Patterns in The Pascal Triangle



Patterns in The Pascal Triangle



Patterns in The Pascal Triangle



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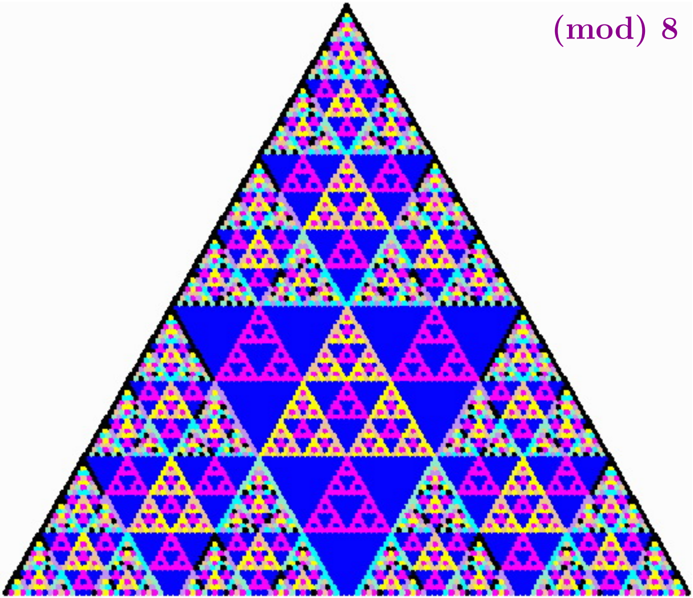
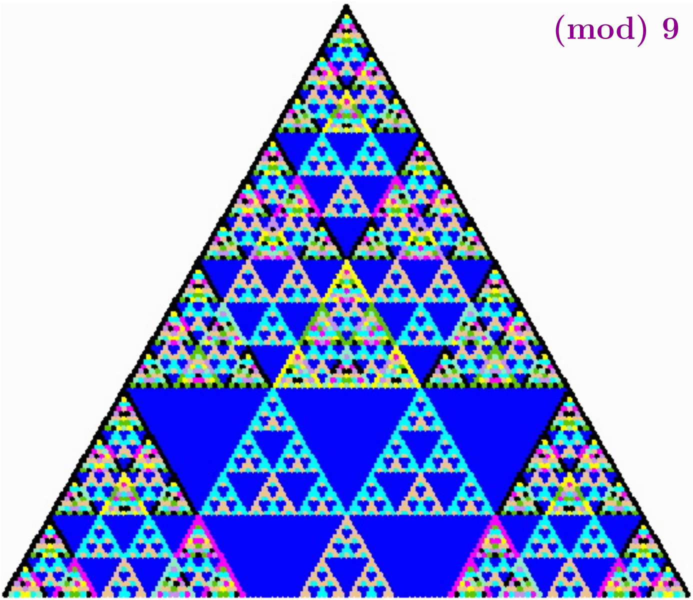


Patterns in The Pascal Triangle



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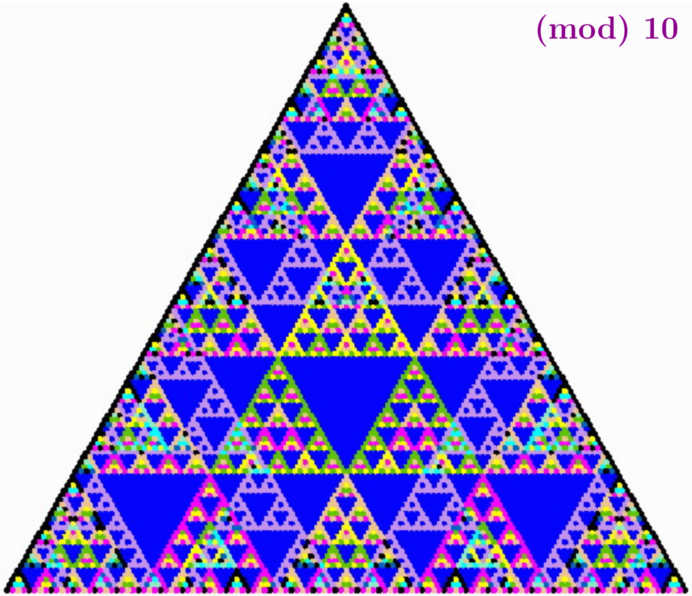
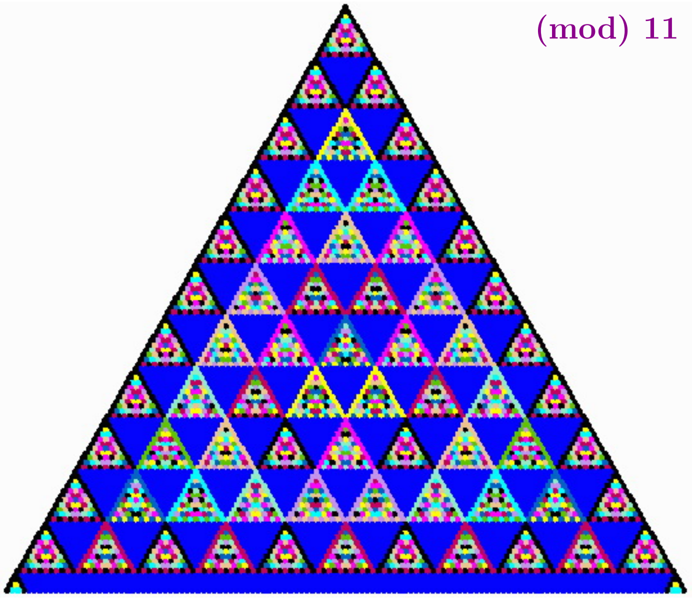
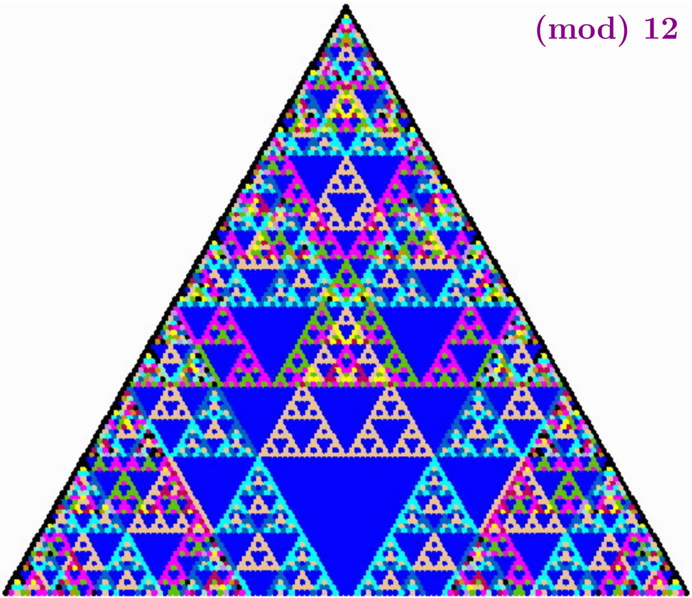


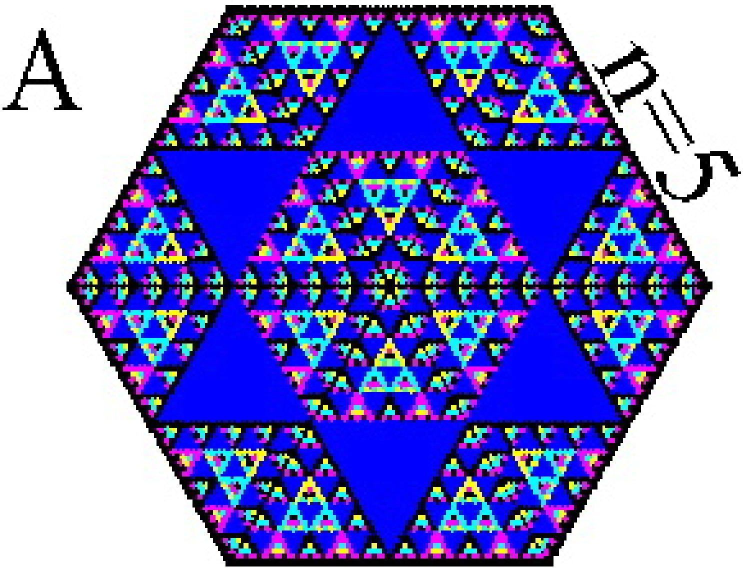

(mod) 11

Patterns in The Pascal Triangle



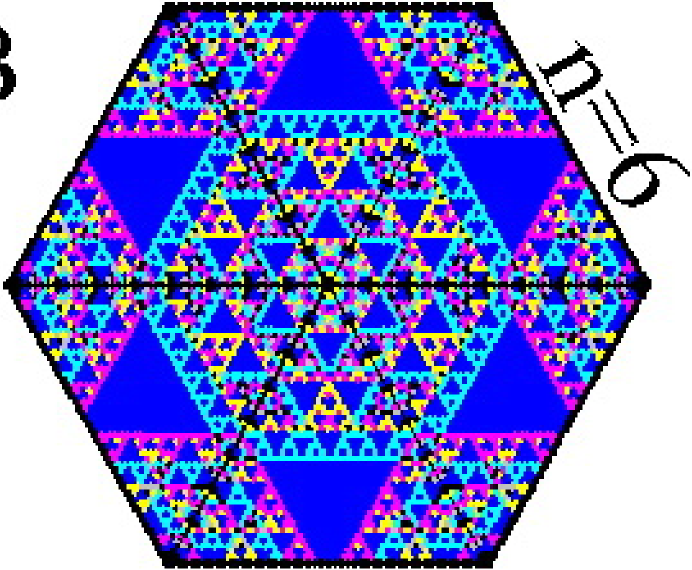
Patterns in The Pascal Triangle

Arrange 6 copies of the Pascal triangle mod n in a hexagon

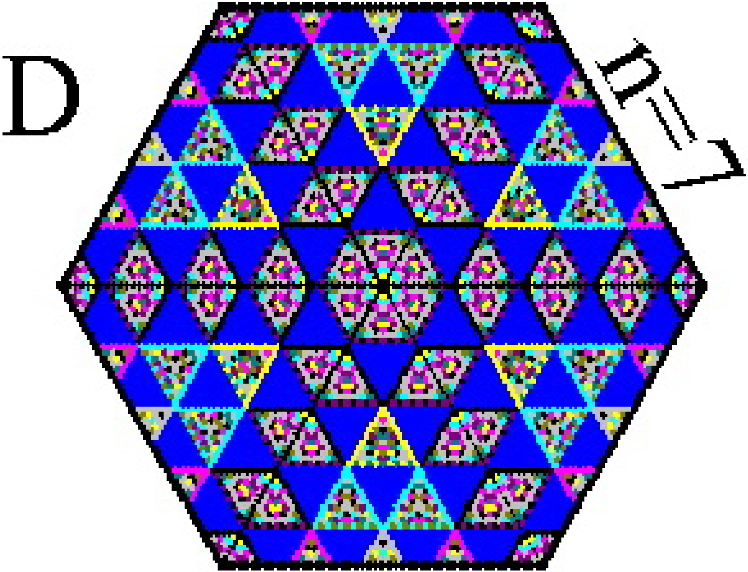

A
n ≡ 5

Patterns in The Pascal Triangle

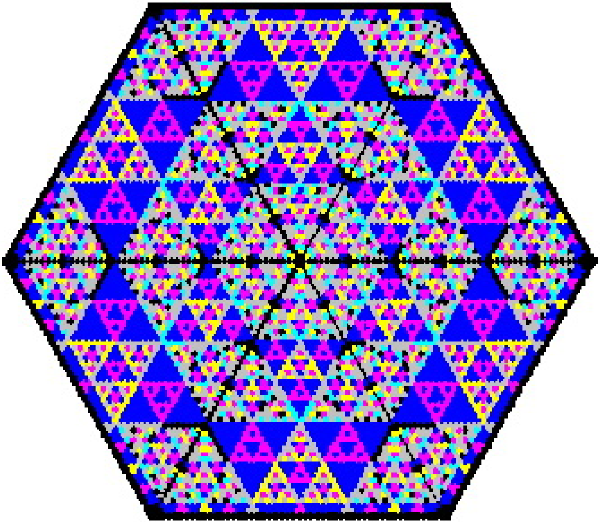
B



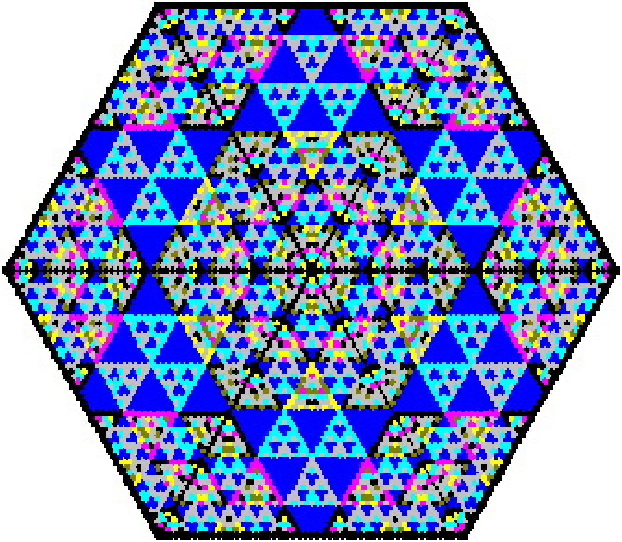
Patterns in The Pascal Triangle



Patterns in The Pascal Triangle

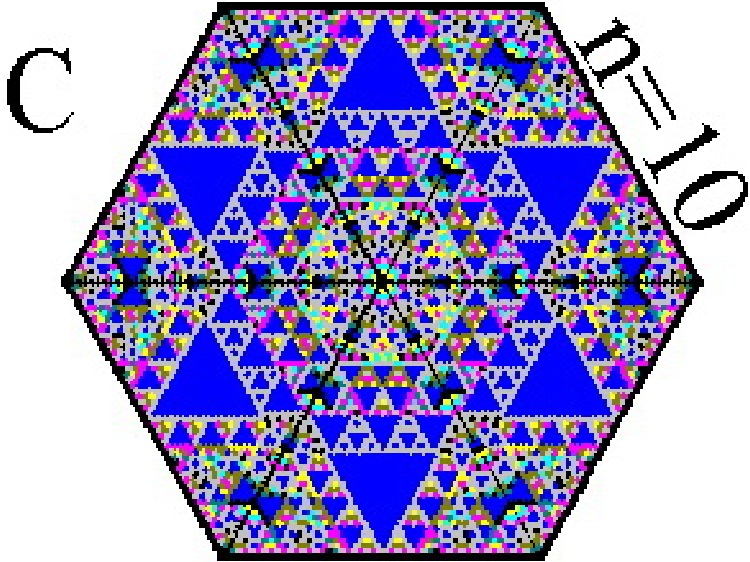


n=8



n=9

Patterns in The Pascal Triangle



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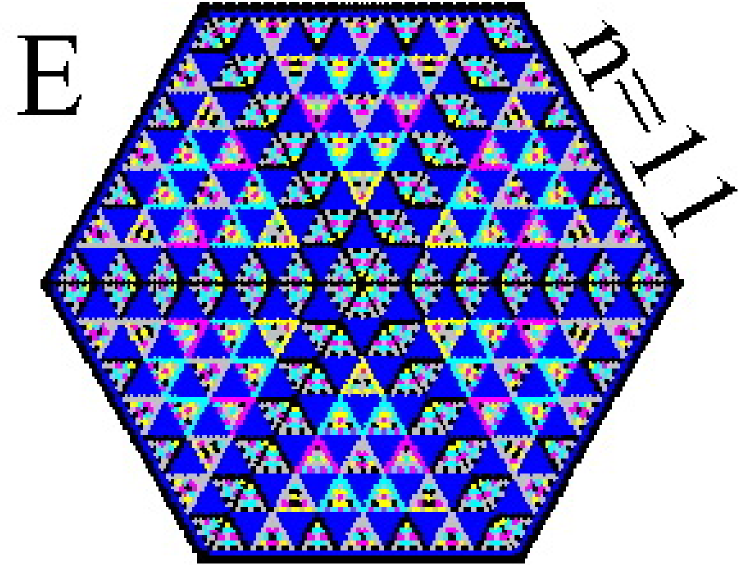

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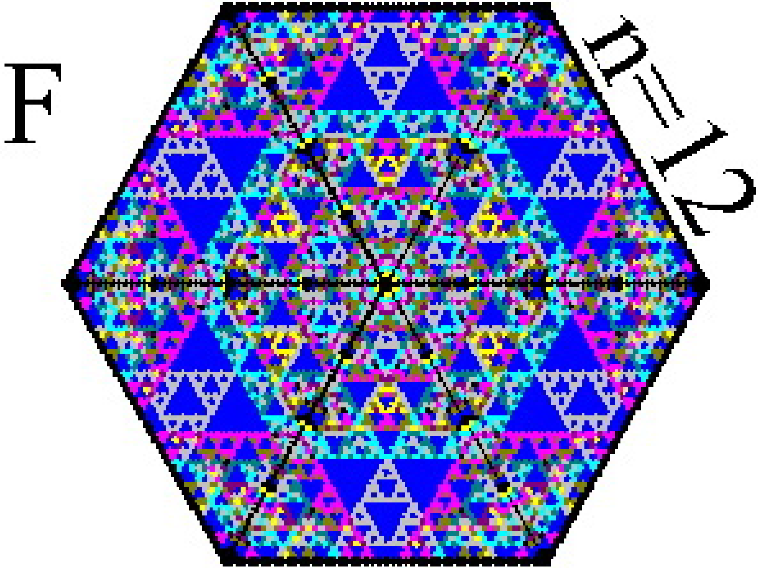


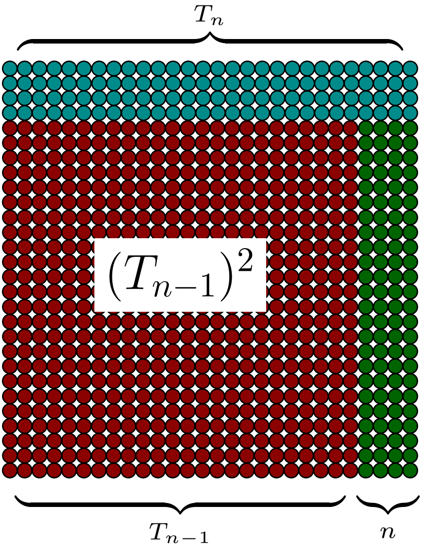
Sum of Cubes

Evaluate

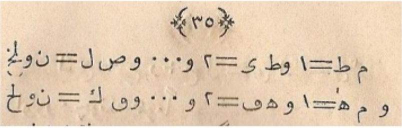
$C_n = 1^3 + 2^3 + 3^3 + \ldots + n^3$

$$
\begin{aligned}
1^3 + 2^3 + \ldots (n-1)^3 + n^3 &= (T_n)^2 \\
&= \left(\frac{n(n+1)}{2}\right)^2
\end{aligned}
$$

Sum of Cubes



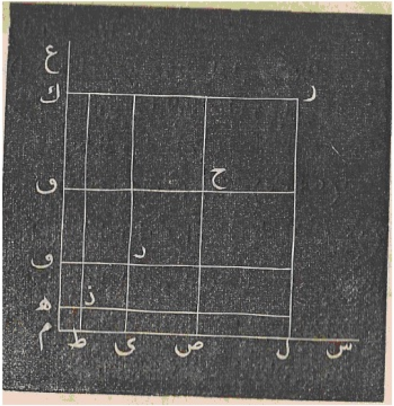
Sum of Cubes: Al-Karaji



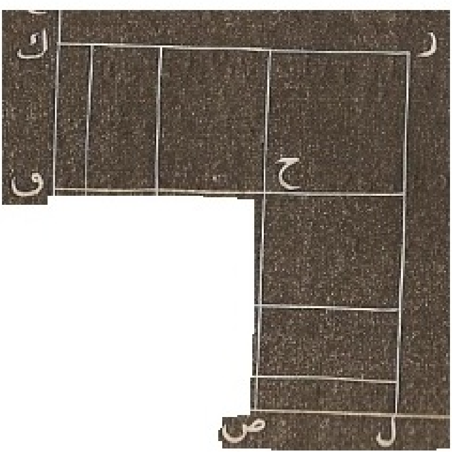
(٣٥)

م ط = ١ وط ى = ٢ و ... وص ل = ن ولخ

و م ه = ١ وه ف = ٢ و ... وق ك = ن ولخ



Sum of Cubes: Al-Karaji



Sum of Cubes: Al-Karaji

(سؤال٨) كرخی نام مؤلف مشهورك كتابندن آلنان اشبو

١×٢+٢×٣+٣×٤+.....+(ن−١) ن

= (ن−١) ن (ن+١) / ٣

مساواتك تحقيق مطلوبدر.

برزمان علمای عربك
معلوملری اولان خواص اعداددن
برمسئله

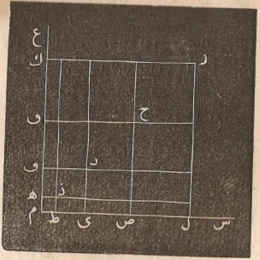
اشبو

١^٣+٢^٣+٣^٣+٤^٣+......+ن^٣

=(١+٢+٣+٤+......+ن)^٢

مساوات ابو بكر محمد بن الحسن الكرخینك تأليفاتندن فخری نامیله معروف اولان جبرومقابله كتابنده وجه آتی اوزره اثبات قلنمشدر. بر برينه عمود م س و م ع خطلری اوزرلرنده م دن بدأ ایله متعاقباً

م ط=١ وط ى=٢ و... وص ل=ن ولچ و م هـ=١ و هـ ف=٢ و... وق ك=ن ولچ اولەرق اخذ و عموماً مثلا ص و ل نقطه‌لرندن م س اوزرينه وكذا بو نقطه‌لرك نظيرلری اولان ق و ك نقطه‌لرندن م ع اوزرينه بر عمود اقامه اولندقده تشكل اولنان ق ح ص ل ر ك ق ساحه‌سی ق ك ف و يا ص ل ك اشعار ايلديكی ن عددينك مكعبنه مساويدر. يعنی



Sum of Cubes: Al-Karaji

١+٢+٣+ ... + ن = ½ ن (ن+١) = لن

و ١+٢+٣+ ... +(ن−١) = ½ ن (ن−١) = لن−١

مساوا تلرندن

ل٢ن − ل٢ن−١ = ¼ ن٢ ((ن+١)٢ − (ن−١)٢) = ن٣

اولمغين

ل٢ن − ل٢ن−١ = ن٣

و ل٢ن−١ − ل٢ن−٢ = (ن−١)٣

و . . . .

و ل٢٢ − ل٢١ = ٢٣

و ل٢١ − ل٢٠ = ١٣

اولوب جمع ايله

ل٢ن = ١٣ + ٢٣ + ٣٣ + . . . + ن٣

بولنور . و دعوى ثابت اولور .