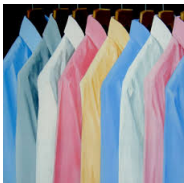# Counting

- Introduction and Applications
- Sum and Product Rule
- The Complement Rule
- Inclusion-Exclusion Principle
- The Pigeon-Hole Principle
- Permutations and Combinations
- Combinatorial Proofs
- Permutation and Combinations with Repetitions

Imdad ullah Khan

# Sum Rule

Suppose you have 9 long sleeves shirts and 8 short sleeves shirts





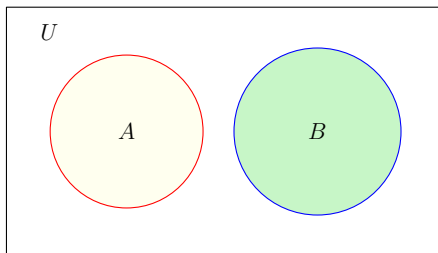How many choices do you have for a shirt?

$$9 + 8 = 17$$

# Sum Rule

### Sum Rule

If a task can be done  <u>either</u>  in one of $n_1$ ways  <u>or</u>  in one of $n_2$ different ways,  then there are $n_1 + n_2$ ways to do the task

# Sum Rule



## Sum Rule

If $A$ and $B$ are disjoint sets, then

$$|A \cup B| = |A| + |B|$$

# Sum Rule

I have 32 students in one section and 35 students in the other section. Suppose I give the grade *A* to one student.

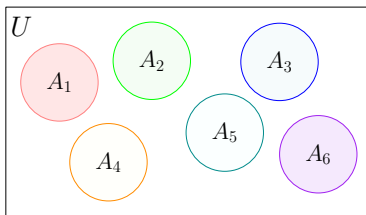<span style="color:red">How many choices do I have in total?</span>

Suppose you have to choose a project from 4 software development projects or from 5 research projects.

<span style="color:red">How many choices do you have?</span>

The sum rule is easy, but identifying the sets or constructing them requires some thinking and trial and error

# Generalized Sum Rule



## Sum Rule

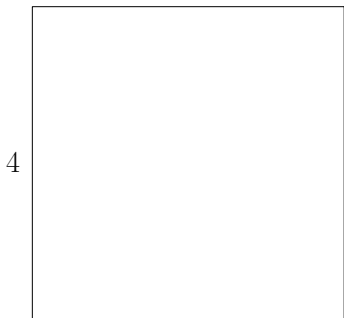If $A_1, A_2, \ldots, A_n$ are **disjoint sets**, then

$$\left| \bigcup_{i=1}^{n} A_i \right| = \sum_{i=1}^{n} |A_i|$$

# Sum Rule

Divide $4'' \times 4''$ square into 16 unit squares by parallel lines

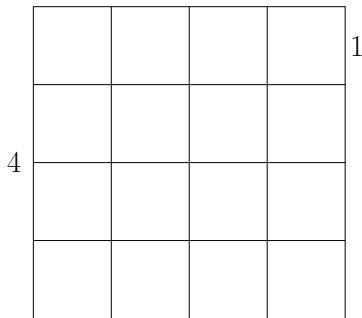**ICP 11-1**    How many squares are there in total?



$4$

# Sum Rule

Divide $4'' \times 4''$ square into 16 unit squares by parallel lines

**ICP 11-1**    How many squares are there in total?

$A_k$ : Number of squares of length $k$

Total number of squares are

$$\left|A_1\right| + \left|A_2\right| + \left|A_3\right| + \left|A_4\right|$$

$\left|A_1\right| = 16 \quad \left|A_2\right| = 9 \quad \left|A_3\right| = ? \; \left|A_4\right| = ?$

# Sum Rule

Suppose I roll red and blue dice!

**ICP 11-2**    In how many outcomes the dice show different values?



12  13  14  15  16
21  23  24  25  26
31  32  34  35  36
41  42  43  45  46
51  52  53  54  56
61  62  63  64  65

# Sum Rule

Suppose I roll red and blue dice!

In how many outcomes the dice show different values?

$A_i$ = set of all outcomes in which

red die shows $i$ and the blue die shows something else

$S$ = set of outcomes where the dice show different values

$$|S| = ?$$

$$|S| = \left| \bigcup_{i=1}^{6} A_i \right| = \sum_{i=1}^{6} |A_i| = ?$$