Counting

- Introduction and Applications
- Sum and Product Rule
- The Complement Rule
- Inclusion-Exclusion Principle
- The Pigeon-Hole Principle
- Permutations and Combinations
- Combinatorial Proofs
- Permutation and Combinations with Repetitions

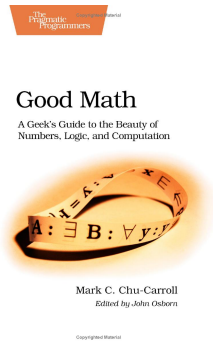
Imdad ullah Khan

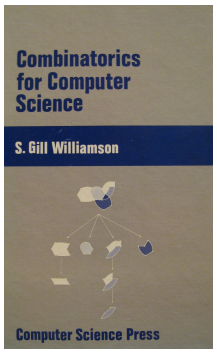
Combinatorics

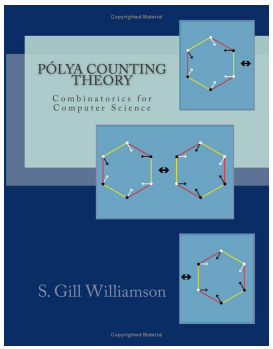
Combinatorics is the study of arrangement of objects

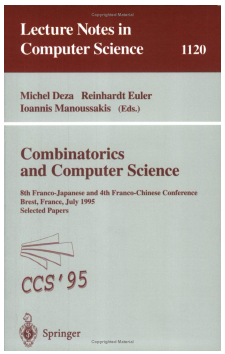
The Art of Counting (enumerative combinatorics)

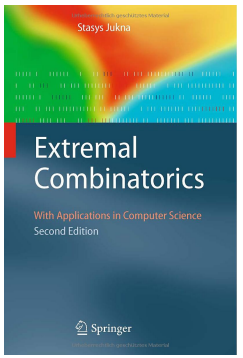
Why Count?

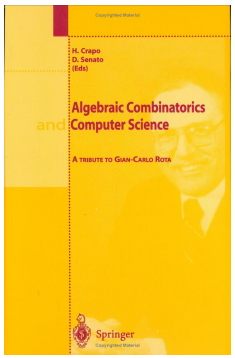






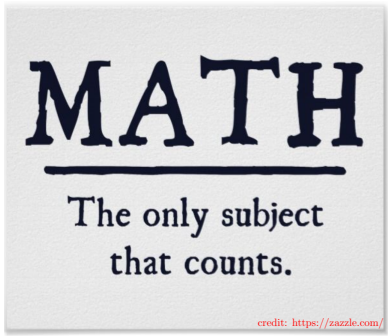






Counting and Combinatorics:

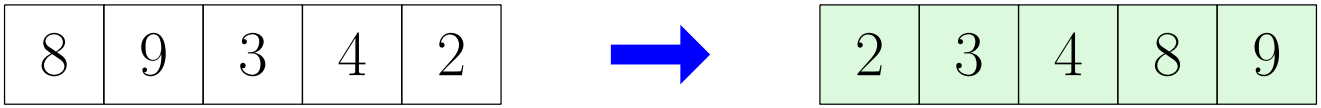
- Calculate the chances of a component failure
- Analyze how long a program takes to finish
- Very useful in designing algorithms



The lottery is a tax on people who flunked math

Monique Lloyd

Why Count?



Algorithm Selection Sort: Sort an array A of n numbers

for $i = 1$ to n **do**
  for $j = i + 1$ to n **do**
    if $A[i] > A[j]$ **then**
      EXCHANGE$(A[i], A[j])$

How many comparison are made?

Why Count?

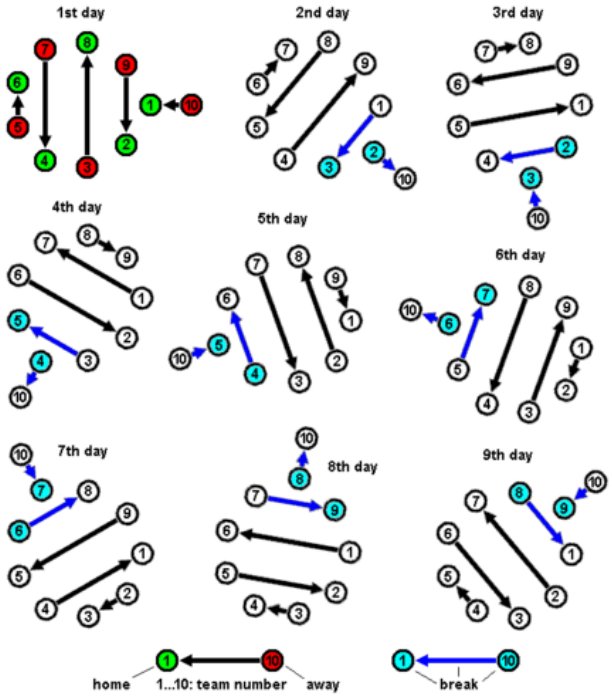
Organizing Tournaments

n teams are participating in Round 1 of a soccer tournament

Every team play every other team exactly once

Each game is refereed by a professional, charging \$1 per game

How many games are played?

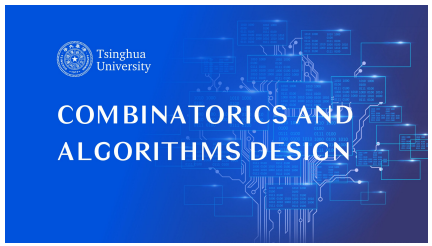


A round-robin tournament with 10 teams (Wikipedia)

Why Count?

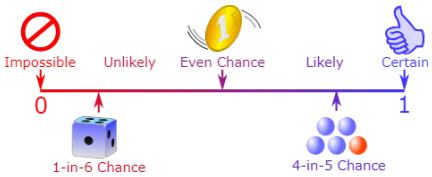
Programming and Algorithm Design

- What should be size of array, other data structures?
- What is the runtime of the algorithm?
- How many arrangements of a specific kind must be generated to run simulations on?



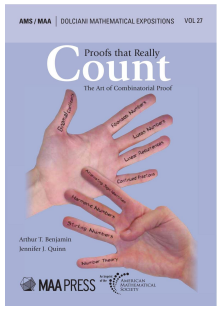
Probability Theory

- What is the runtime of a randomized algorithm?
- What are the chances that a network link will fail?



Combinatorial Proofs

- Elegant counting based proofs



Why Count?

- How many bit strings of length n are there?
- How many paths between two nodes on the Internet?
- How many valid IP addresses are there?
- How many steps are required to do sorting?
- How many valid 6 character passwords are possible?
- How many ways are there to buy 13 different bagels from a shop that sells 17 types?
- How many bit strings of length 11 contain a streak of one type of bit of exact length 7?
- How many ways can a hiring service match 13 jobs to 17 applicants?
- How many arrangements are there of a deck of cards?

Counting Techniques

We will learn basic counting techniques to answer this type of questions and apply them in applications mentioned above

- Sum Rule and Product Rule
- The Complement principle
- Inclusion-exclusion principle
- Pigeon-hold principle
- Permutations and Combinations
- Permutations and Combinations with Repetitions