Induction

- Principle of Mathematical Induction
- Proofs by Induction
- Strong Induction
- Well Ordering Principle

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Proof

Proof: an argument that convincingly demonstrate truth of a statement

In Computer Science and Engineering

- Prove that the algorithm is correct
- Prove that algorithm has a particular running time
- Data structure proofs often lead to efficient and simpler algorithm

In general, working with proofs develops useful habits in thinking

- Working with precise notions
- Exactly formulating statements
- Paying attention to all possibilities

Induction

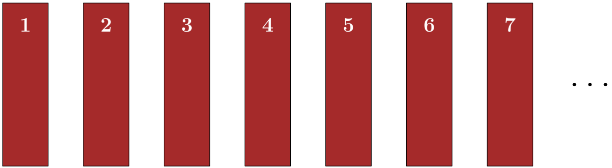
Induction is useful to prove statements about all positive integers,

$$\sum_{i=0}^{n} i \; = \; 1 + 2 + 3 + \ldots + (n-1) + n \; = \; \frac{n(n+1)}{2}$$

Number of subsets of an n-element set is 2^n

Principle of Induction

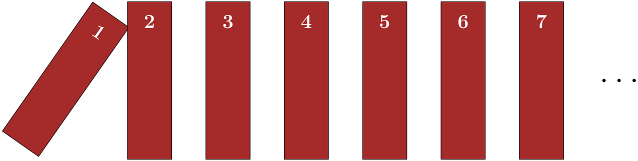
Suppose we have an infinite number of dominoes



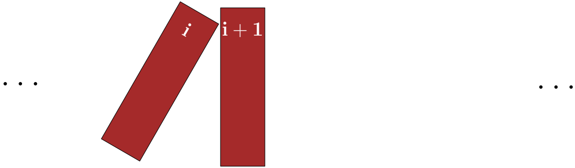
Principle of Induction

Suppose two rules/facts are given

- **Fact 1:** First domino falls



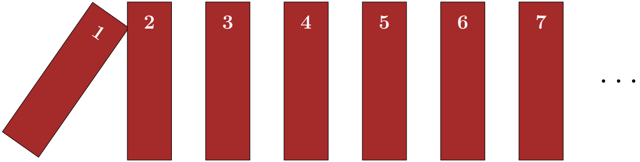
- **Fact 2:** If a domino falls, then the next domino also falls



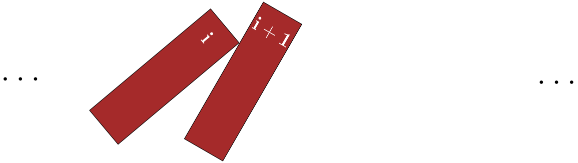
Principle of Induction

Suppose two rules/facts are given

1. **Fact 1:** First domino falls

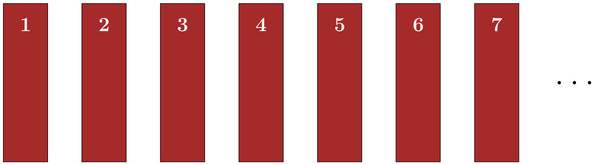


2. **Fact 2:** If a domino falls, then the next domino also falls



Principle of Induction

Applying both **Fact 1** and **Fact 2**



Principle of Induction

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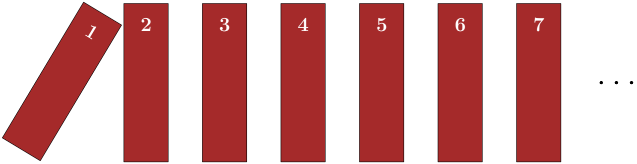
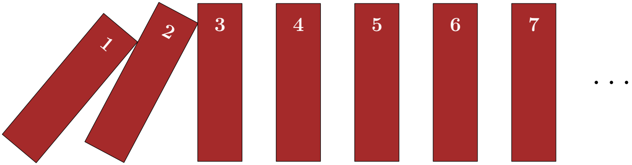


Principle of Induction

Applying both **Fact 1** and **Fact 2**



Principle of Induction

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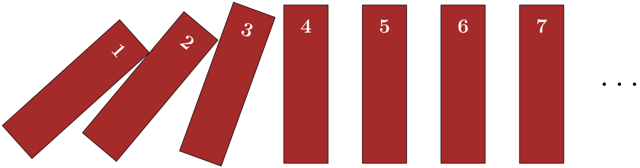

Principle of Induction

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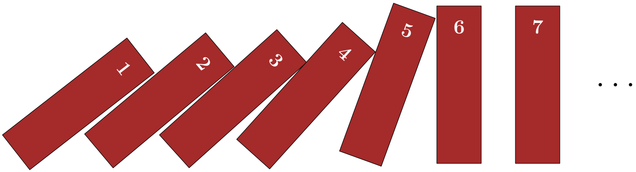
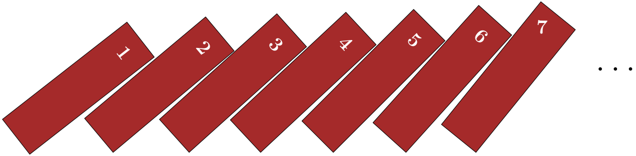


Principle of Induction

Applying both **Fact 1** and **Fact 2**



Can we conclude that all dominoes fall?

- Yes
- No

Proof by Induction

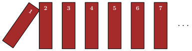
A proposition about non-negative integers, $\forall\ n\ P(n)$ is a sequence of propositions (dominoes)

$$P(0), P(1), P(2), \ldots, P(n), P(n+1), \ldots$$

Establish two facts

- Prove $P(0)$

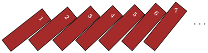
  the first domino falls



- Prove $\forall k \geq 0, P(k) \rightarrow P(k+1)$

  if a domino falls, then the next domino also falls

Conclude that $P(n)$ is true for all n



Principle of Mathematical Induction

$$\Big[\, P(0)\ \wedge\ \forall k \geq 0\, \big[P(k) \rightarrow P(k+1)\big]\,\Big]\ \longrightarrow\ \forall n \geq 0\ P(n)$$