Relations

- Relations: Definition and Notation
- Properties of Relations
- Combining Relations
- Operations on Relations: Projection and Join
- Equivalence Relations and Equivalence Classes
- Partial Order

Imdad ullah Khan

Relations on a Set

Relation

A (binary) relation from X to Y is a subset of $X \times Y$

Relation on a Set

A (binary) relation on a set X is a subset of $X \times X$ (relation from X to X)

Projection on n-tuple

Projection

The projection $P_{i_1,i_2,\ldots,i_m}$ maps the n-tuple $(a_1, a_2, \ldots, a_n)$ to the m-tuple $(a_{i_1}, a_{i_2}, \ldots, a_{i_m})$, where $m \leq n$

Projection $P_{i_1,i_2,\ldots,i_m}$ keeps the m coordinates $a_{i_1}, a_{i_2}, \ldots, a_{i_m}$ of an n-tuple and removes the remaining $(n - m)$ other coordinates

Apply $P_{1,4}$ to $\{(a_1, a_2, a_3, a_4, a_5)\}$

we get $\{(a_1, a_4)\}$ projecting on 1st and 4th coordinates

Apply $P_{2,3}$ to $\{(Ahmad, 41234, CS, 3.5)\}$

we get $\{(41234, CS)\}$ projecting on 2nd and 3rd coordinates

Projection on Relations

Apply projection $P_{\cdot,\cdot,\ldots}$ to all n-tuples in an n-ary relation

Apply $P_{2,3,4}$ to

Name	ID	Dept.	CGPA
Ahmad	43211	CS	3.2
Ali	43389	CS	2.7
Fatima	43324	Phy	2.9
Ali	43196	EE	3.6
Ahmad	43201	CS	3.4

We get

ID	Dept.	CGPA
43211	CS	3.2
43389	CS	2.7
43324	Phy	2.9
43196	EE	3.6
43201	CS	3.4

Projection on Relations

Apply projection $P_{\cdot,\cdot,\ldots}$ to all n-tuples in an n-ary relation

ICP 6-23

Apply $P_{2,3}$ to

Name	ID	Dept.	CGPA
Ahmad	43211	CS	3.2
Ali	43389	CS	2.7
Fatima	43324	Phy	2.9
Ali	43196	EE	3.6
Ahmad	43201	CS	3.4

We get

ID	Dept.
43211	CS
43389	CS
43324	Phy
43196	EE
43201	CS

Projection

Sometime a projection may result in fewer records

Apply $P_{1,3}$ to

Name	ID	Dept.	CGPA
Ahmad	43211	CS	3.2
Ali	43389	CS	2.7
Fatima	43324	Phy	2.9
Ali	43196	EE	3.6
Ahmad	43201	CS	3.4

We get

Name	Dept.
Ahmad	CS
Ali	CS
Fatima	Phy
Ali	EE
Ahmad	CS

which is the same as

Name	Dept.
Ahmad	CS
Ali	CS
Fatima	Phy
Ali	EE

ICP 6-24 In what scenarios we may get fewer records ?

ICP 6-25 Which coordinate if included will not result in loss of records?

Join

- R: a relation of degree m
- S: a relation of degree n
- $J_p(R, S)$ is a relation of degree $m + n - p$, $(p \leq m \wedge p \leq n)$ containing

$$(\overbrace{a_1, a_2, \ldots, a_{m-p}\ c_1, c_2, \ldots, c_p}^{\in R}, \underbrace{b_1, b_2, \ldots, b_{n-p}}_{\in S})$$

- $(a_1, a_2, \ldots, a_{m-p}, c_1, c_2, \ldots, c_p) \in R$
- $(c_1, c_2, \ldots, c_p, b_1, b_2, \ldots, b_{n-p}) \in S$

$J_p(R, S)$ "joins" all pairs of m-tuple $\mathbf{x} \in R$ and n-tuple $\mathbf{y} \in S$ if the last p coordinates of $\mathbf{x}$ and the first p coordinates of $\mathbf{y}$ are the same

Join

CS Course Assignment		
Name	Course	Code
Imdad	DM	210
Junaid	DS	310
Arif	SP	412
Basit	CP	211

Sem Course Schedule			
Code	Room	Day	Time
210	M-11	Mon	8-10 AM
211	Q-02	Wed	8-10 AM
310	M-09	Tue	9-11 AM
211	N-06	Thu	9-11 AM
412	M-11	Tue	12-1 PM

$J_1(CS\_Assignment, Sem\_Schedule)$

CS Faculty Schedule					
Name	Course	Code	Room	Day	Time
Imdad	DM	210	M-11	Mon	8-10 AM
Junaid	DS	310	M-09	Tue	9-11 AM
Arif	SP	412	M-11	Tue	12-1 PM
Basit	CP	211	Q-02	Wed	8-10 AM
Basit	CP	211	N-06	Thu	9-11 AM

Join

Join generalizes intersection and cross product

R: a relation of degree n

S: a relation of degree n

$$J_n(R, S) = R \cap S$$

$$J_0(R, S) = \text{“ } R \times S \text{ ”}$$