Relations

- Relations: Definition and Notation
- Properties of Relations
- Combining Relations
- Operations on Relations: Projection and Join
- Equivalence Relations and Equivalence Classes
- Partial Order

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Ordered Tuples and Cartesian Product

The ordered n-tuple $(a_1, a_2, \ldots, a_n)$ is an ordered collection of n objects

Ordered 2-tuples ($n = 2$) are called ordered pairs

Cartesian product of sets A and B is the set of all ordered pairs (x, y), where $x \in A$ and $y \in B$

$S = \{x_1, x_2, x_3\}$ and $G = \{A, B\}$

$$S \times G = \Big\{(x_1, A), (x_1, B), (x_2, A), (x_2, B), (x_3, A), (x_3, B)\Big\}$$

$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$: the Cartesian plane or Euclidean Plane

▷ Cartesian product of $\mathbb{R}$ (x-axis) and $\mathbb{R}$ (y-axis)

Relations

Function

Let X and Y be two sets.

A function f maps **each** element of X to **exactly one** element of Y

Function

A function from X to Y is a subset of $X \times Y$, such that for every $a \in X$, f contains exactly one ordered pair with the first coordinate a

Relation

A relation from X to Y is a subset of $X \times Y$, ~~such that for every $a \in X$, f contains exactly one ordered pair with the first coordinate a~~

Relations

Binary Relation

A relation from X to Y is a subset of $X \times Y$

n-ary Relation

Let $A_1, A_2, \ldots, A_n$ be sets.

An n-ary relation is a subset of $A_1 \times A_2 \times \ldots \times A_n$

- The sets $A_1, A_2, \ldots, A_n$ are called **domains** of the relation
- n is called the **degree** of the relation

Relations: Example

Database			
Name	ID	Dept.	CGPA
Ahmad	43211	CS	3.2
Ali	43389	CS	2.7
Usman	43324	Phy	2.9
Ali	43196	CS	3.6
Fatima	43201	EE	3.4

$$\text{Database} \subseteq \{\text{Names}\} \times \{\text{IDs}\} \times \{\text{Depts}\} \times \{x : x \in \mathbb{R}, 0 \leq x \leq 4\}$$

Relations: Example

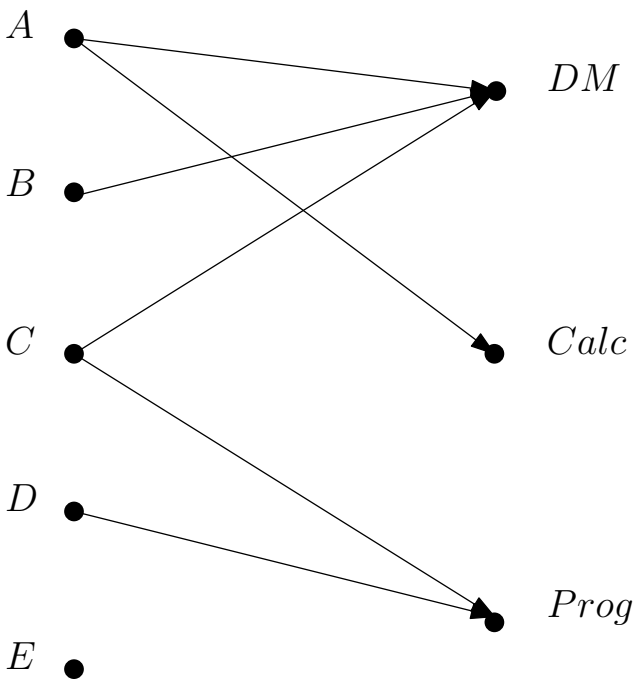


Figure: Registration

$$\text{Registration} \subseteq \{\text{ Students}\} \times \{\text{ Courses}\}$$

$$\text{Registration} = \{(A, DM), (A, Calc), (B, DM), (C, DM), (C, Prog), (D, Prog)\}$$

Relations: Notation

$$X = \underbrace{\{A, B, C, D, E\}}_{Students} \qquad \underbrace{Y = \{Calc, DM, Prog\}}_{Courses}$$

$$\mathbf{R} := \Big\{(A, DM), (A, Calc), (B, DM), (C, DM), (C, Prog), (D, Prog)\Big\}$$

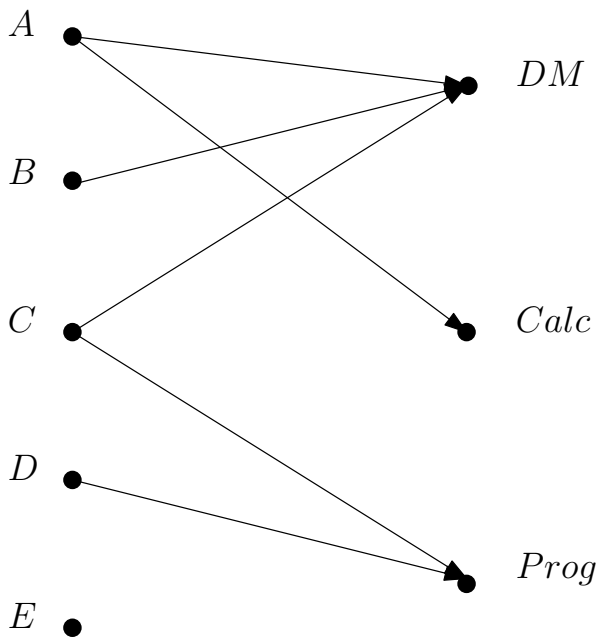
is a relation

- $(A, DM) \in \mathbf{R}$ or $A\ \mathbf{R}\ DM$ or $\mathbf{R}(A, DM) = 1(true)$
- $(B, Calc) \notin \mathbf{R}$
- Note that E is not related to any
- A and C are related two elements of Y each

Relations: Graphical Representation

$$X = \underbrace{\{A, B, C, D, E\}}_{Students} \qquad \underbrace{Y = \{Calc, DM, Prog\}}_{Courses}$$

$$R := \{(A, DM), (A, Calc), (B, DM), (C, DM), (C, Prog), (D, Prog)\}$$



Relations Applications

- Business and its telephone numbers
- Students and their GPA's
- Instrutors and their courses
- Computer language and its valid statements
- Computers connected via a network
- Viable schedule for different activities of a project
- **Relational** Database Management System (RDBMS)

Relations on a Set

Relation

A (binary) relation from X to Y is a subset of $X \times Y$

Relation on a Set

A (binary) relation on a set X is a subset of $X \times X$ (relation from X to X)

Relations on a Set

$A = \{1, 2, 3, 4, 6\}$

Relation on A:

$R := \{(x, y) \mid x \text{ divides } y\}$

$(1, 1) \in^? R$ Yes

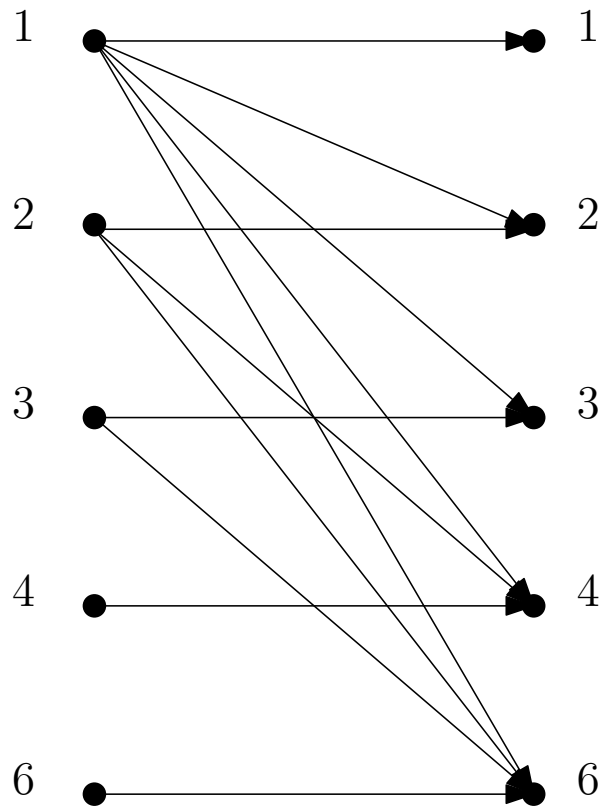
$(1, 2) \in^? R$ Yes

$(2, 3) \in^? R$ No

ICP 6-1 $(3, 6) \in^? R$

ICP 6-2 $(4, 6) \in^? R$

ICP 6-3 What is $|R|$?



Relations on a Set

Relations on the set of integers

- $R_1 = \{(a, b) \mid a \leq b\}$
- $R_2 = \{(a, b) \mid a > b\}$
- $R_3 = \{(a, b) \mid a = b \text{ or } a = -b\}$
- $R_4 = \{(a, b) \mid a = b\}$
- $R_5 = \{(a, b) \mid a = b + 1\}$
- $R_6 = \{(a, b) \mid a + b \leq 3\}$

ICP 6-4 Check whether or not the ordered pair is in the relation

	R_1	R_2	R_3	R_4	R_5	R_6
$(1, 1)$	✓	✗				
$(1, 2)$	✓	✗				
$(2, 1)$	✗	✓				
$(1, -1)$	✗	✓				
$(2, 2)$	✓	✗				