Functions

- Ordered tuples and Cartesian Product
- Function and Representations
- Types of Functions
- Composition and Inverse of Function
- Numeric Functions

IMDAD ULLAH KHAN

Ordered Tuples and Cartesian Product: Review

- Ordered n-tuple $(a_1, a_2, \ldots, a_n)$ is an ordered collection of n objects
- $(a_1, a_2, \ldots, a_n) = (b_1, b_2, \ldots, b_n)$ means $a_i = b_i$ for $1 \leq i \leq n$
- Ordered 2-tuples $(n = 2)$ are called ordered pairs
- Cartesian product of sets A and B, $A \times B$ is
  the set of all ordered pairs (x, y), where $x \in A$ and $y \in B$
- Cartesian product can be generalized to that of more than 2 sets
- $\left|A_1 \times A_2 \times \ldots \times A_n\right| = |A_1| \times |A_2| \times \ldots \times |A_n|$

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- Crazy Professor's map: $\Big\{(x_1, F), (x_2, F), (x_3, F), (x_4, F), (x_5, F)\Big\}$

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- Reasonable map: $\Big\{(x_1, C), (x_2, A), (x_3, B), (x_4, B), (x_5, A)\Big\}$

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$f : X \mapsto Y$ is a subset of $X \times Y$, such that for every $x \in X$, f contains exactly one ordered pair with first component x

Function Graphical Representation

- $f : X \mapsto Y$ can be represented graphically (bipartite graphs)
- '*Draw*' sets X and Y. For $f(x) = y$ draw arrow from $x \in X$ to $y \in Y$

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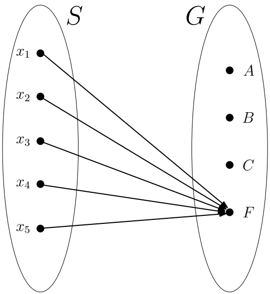

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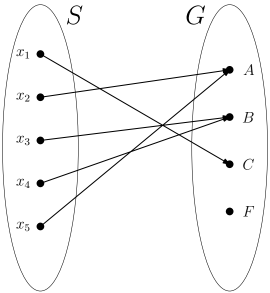

Functions: Basic Terms

Let X and Y be two sets.

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Let $f : X \mapsto Y$ and let $f(x) = y$

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- y is the image of x
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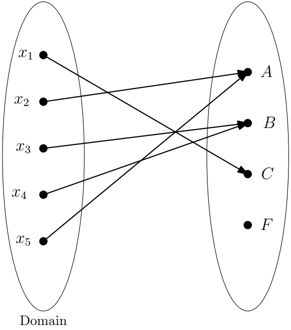
- Range of f:
  set of images of all elements of X



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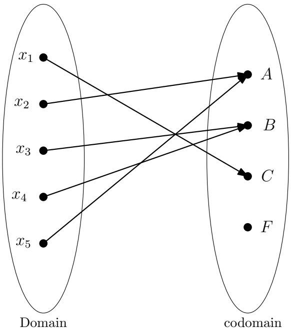

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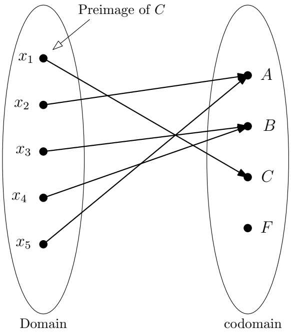

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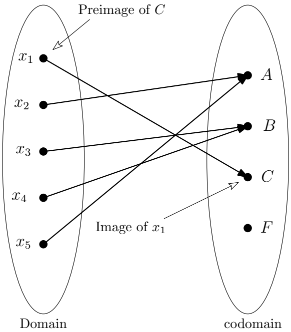

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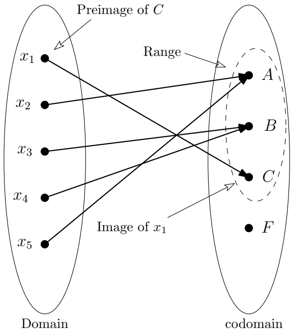

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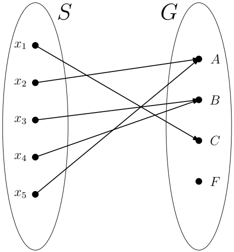

ICP 5-5 Range of $f = \{A, B, C\}$? a) True b) False

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- $f : S \mapsto G := \Big\{(x_1, F), (x_2, F), (x_3, F), (x_4, F), (x_5, F)\Big\}$

Function: Basic Terms

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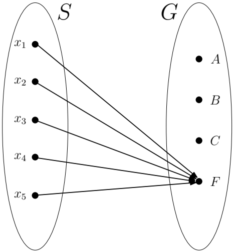

ICP 5-6 Range of $f = \{F\}$? a) True b) False

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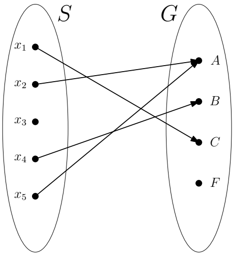

ICP 5-7 Range of $f = \{A, B, C\}$? a) True b) False

Function: Basic Terms

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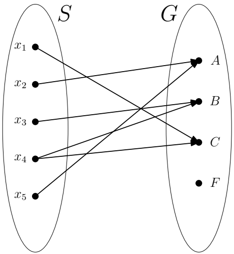

ICP 5-8 Range of $f = \{A, B, C\}$? a) True b) False

Functions: Mapping Rule

For large domains difficult to describe functions with graphs or sets

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Functions can be specified by a mapping rule (formula)

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Let $S = \underbrace{\{x_1, x_2, x_3, x_4, x_5\}}_{Students}, \qquad G = \underbrace{\{A, B, C, F\}}_{Grades}$

$f : S \mapsto G$ is defined as follows:

$$f(x_i) = \begin{cases} A & if\ score(x_i) \geq 90\% \\ B & if\ 80\% \leq\ score(x_i) < 90\% \\ C & if\ 60\% \leq\ score(x_i) < 80\% \\ F & if\ score(x_i) < 60\% \end{cases}$$

Functions: Mapping Rule

Let $f : \mathbb{Z} \mapsto \mathbb{Z}$ be given as

$$f(x) = x^2$$

- Domain of f
- Codomain of f
- Image of -3 and 5
- Preimage of $25, 4, -3, 30$
- Range of f

Functions: C++/JAVA

```
function SIGN(x)                    ▷ returns int
    if x < 0 then
        return −1
    else if x = 0 then
        return 0
    else
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function SIGN(x)                ▷ returns int
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```

- Domain of funct()
- Codomain of funct()
- Image of $-3, 2, 5, 0$
- Preimage of $0, 4, 1, -1$
- Range of funct()
- Functionality of funct()

Functions: Summary

- $f : X \mapsto Y$ maps **each** element of X to **exactly one** element of Y

Let $f : X \mapsto Y$ and let $f(x) = y$

- X is the domain of f
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- y is the image of x
- x is the pre-image of y
- Range of f: set of images of all elements of X

- Functions can be represented by
  - Listing set of all (pre-image, image) ordered pairs
  - Bipartite Graph
  - Mapping Rule or Algebraic Expression
  - Programming Code