Functions

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Ordered tuples

The ordered n-tuple $(a_1, a_2, \ldots, a_n)$ is an ordered collection of n objects

- Unlike sets order matters here
- Notation: $(a_1, a_2, \ldots, a_n)$ unlike $\{a_1, a_2, \ldots, a_n\}$
- Repetition matters too
- More like C++/Java arrays, except for type restriction is not applied

Ordered tuples

The ordered n-tuple $(a_1, a_2, \ldots, a_n)$ is an ordered collection of n objects

$$(a_1, a_2, \ldots, a_n) = (b_1, b_2, \ldots, b_n) \text{ means } a_i = b_i \text{ for } 1 \leq i \leq n$$

- $(1, 2, 9) \;=\; (1, 2, 9)$
- $(1, apple, 8, car) \;\neq\; (1, apple, car, 8)$
- $(3, 5, 7) \;\neq\; (3, 5, 7, 11)$

Which one of the following is true?

ICP 5-1 $(3, 5, 7, 11) \;=\; (3, 7, 5, 11)$ a) True b) False

ICP 5-2 $(3, 5, 5, 7) \;\neq\; (3, 5, 7)$ a) True b) False

ICP 5-3 $(3, 5, 5, 7) \;=\; (3, 5, 7, 5)$ a) True b) False

ICP 5-4 $(x, A) \;\neq\; (A, x)$ a) True b) False

Cartesian Product

Ordered 2-tuples ($n = 2$) are called ordered pairs

Cartesian product of sets A and B is the set of all ordered pairs (x, y), where $x \in A$ and $y \in B$

$$A \times B \;=\; \big\{(x, y) \mid (x \in A) \wedge (y \in B)\big\}$$

▷ Also known as the cross product

$S = \{x_1, x_2, x_3\}$ and $G = \{A, B\}$

$$S \times G \;=\; \Big\{(x_1, A), (x_1, B), (x_2, A), (x_2, B), (x_3, A), (x_3, B)\Big\}$$

$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$: the Cartesian plane or Euclidean Plane

▷ Cartesian product of $\mathbb{R}$ (x-axis) and $\mathbb{R}$ (y-axis)

Cartesian Product

$$X = \{1, 2\}, \qquad Y = \{a, b, c\}$$

- $X \times Y \; = \; \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$
- $Y \times X \; = \; \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$
- $X \times X \; = \; \{(1, 1), (1, 2), (2, 1), (2, 2)\}$
- $X \times \emptyset \; = \; \emptyset$
- $\emptyset \times X \; = \; \emptyset$

ICP 5-5 Generally, $A \times B \; = \; B \times A$? a) True b) False

$A \times B \; \neq \; B \times A$ unless $A = \emptyset$ or $B = \emptyset$ or $A = B$

Cartesian Product of more than two sets

Readily generalize to more than 2 sets

$A = \{p, q\}, \quad B = \{i, j\}, \quad C = \{4, 5, 6\}$

$$A \times B \times C = \Big\{(p,i,4),(p,i,5),(p,i,6),(p,j,4),(p,j,5),(p,j,6),$$
$$(q,i,4),(q,i,5),(q,i,6),(q,j,4),(q,j,5),(q,j,6)\Big\}$$

Cardinality of Cartesian product is the product of cardinalities

$$|A_1 \times A_2 \times \ldots \times A_n| = |A_1| \times |A_2| \times \ldots \times |A_n|$$

Ordered Tuples and Cartesian Product: Summary

- Ordered n-tuple $(a_1, a_2, \ldots, a_n)$ is an ordered collection of n objects
- $(a_1, a_2, \ldots, a_n) = (b_1, b_2, \ldots, b_n)$ means $a_i = b_i$ for $1 \leq i \leq n$
- Ordered 2-tuples ($n = 2$) are called ordered pairs
- Cartesian product of sets A and B, $A \times B$ is
  the set of all ordered pairs (x, y), where $x \in A$ and $y \in B$
- Cartesian product can be generalized to that of more than 2 sets
- $\left|A_1 \times A_2 \times \ldots \times A_n\right| = |A_1| \times |A_2| \times \ldots \times |A_n|$