Set Theory

- Sets: Definition, Universal Set, Complement, Cardinality
- Subset and Power Set
- Sets Operations
- Set Equality
- Characteristic Vectors: Sets as Bit-Vectors
- Multisets

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Set Equality using Identities

Two sets are equal if and only if they have the same elements

$$A = B \quad \text{means} \quad \forall x \, (x \in A \leftrightarrow x \in B)$$

- To prove two sets A and B to be equal
- Start with one set (say A) and replace it with an equal set
- These established equalities between sets are called "set identities"
- Continue doing this until we get the set B

Set Identities

Identity	Name
$A \cup \emptyset = A$ $A \cap U = A$	Identity Laws
$A \cap \emptyset = \emptyset$ $A \cup U = U$	Domination Laws
$A \cup A = A$ $A \cap A = A$	Idempotent Laws

Set Identities

Identity	Name
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative Laws
$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$	Associative Laws
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive Laws

Set Identities

Identity	Name
$\overline{(\overline{A})} = A$	Double Complement Law
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement Laws
$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$	De Morgan's Laws

Set Identities: Demonstration

$U = \{1, 2, 3, 4, 5, 6\} \qquad A = \{2, 3, 5\} \qquad B = \{2, 3, 4\}$

- $\overline{B} = \{1, 5, 6\}$
- $\overline{A} = \{1, 4, 6\}$
- $\overline{(\overline{A})} = \{2, 3, 5\}$
- $A \cup \overline{A} = \{1, 2, 3, 4, 5, 6\}$
- $A \cap \overline{A} = \{\}$

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- $A \cup \overline{A} = \{1,2,3,4,5,6\}$
- $A \cap \overline{A} = \{\}$
- $A \cap B = \{2,3\}$
- $A \cup B = \{2,3,4,5\}$
- $\overline{A \cap B} = \{1,4,5,6\}$
- $\overline{A} \cup \overline{B} = \{1,4,5,6\}$
- $\overline{A \cup B} = \{1,6\}$
- $\overline{A} \cap \overline{B} = \{1,6\}$

Set Identities: Demonstration

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- $\overline{B} = \{1, 5, 6\}$
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- $A \cap B = \{2, 3\}$
- $A \cup B = \{2, 3, 4, 5\}$
- $\overline{A \cap B} = \{1, 4, 5, 6\}$
- $\overline{A} \cup \overline{B} = \{1, 4, 5, 6\}$
- $\overline{A \cup B} = \{1, 6\}$
- $\overline{A} \cap \overline{B} = \{1, 6\}$

- **ICP 4-23** $\quad B \cap \overline{B} = ?$
- **ICP 4-24** $\quad B \cup \overline{B} = ?$
- **ICP 4-25** $\quad \overline{(\overline{B})} = ?$

Set Equalities using Identities

Show using set identities that

$$\overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$$

Set Equalities using Identities

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$$LHS = \overline{A \cup B \cup C}$$

Set Equalities using Identities

Show using set identities that

$$\overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$$

$$LHS = \overline{A \cup B \cup C}$$

$$= \overline{A} \cap \overline{(B \cup C)} \qquad \text{DeMorgan's Law}$$

Set Equalities using Identities

Show using set identities that

$$\overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$$

$$
\begin{aligned}
LHS &= \overline{A \cup B \cup C} & \\
&= \overline{A} \cap \overline{(B \cup C)} & \text{DeMorgan's Law} \\
&= \overline{A} \cap (\overline{B} \cap \overline{C}) & \text{DeMorgan's Law}
\end{aligned}
$$

Set Equalities using Identities

Show using set identities that

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LHS &= \overline{A \cup B \cup C} & \\
&= \overline{A} \cap \overline{(B \cup C)} & \text{DeMorgan's Law} \\
&= \overline{A} \cap (\overline{B} \cap \overline{C}) & \text{DeMorgan's Law} \\
&= \overline{A} \cap \overline{B} \cap \overline{C} & \text{Associative Law}
\end{aligned}$$

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Show using set identities that

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LHS &= \overline{A \cup B \cup C} & \\
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&= \overline{A} \cap (\overline{B} \cap \overline{C}) & \text{DeMorgan's Law} \\
&= \overline{A} \cap \overline{B} \cap \overline{C} & \text{Associative Law} \\
&= RHS &
\end{aligned}$$

Set Equalities using Membership Table

Two sets are equal if and only if they have the same elements

$$A = B \quad \text{means} \quad \forall x \, (x \in A \leftrightarrow x \in B)$$

- To prove two sets A and B to be equal
- We directly prove the above definition of equality
- For *every element* $x \in U$, we prove that it is either both in A and B or none of them
- When A and B are defined in terms of sets operations on other sets, *every element* $x \in U$ means all types of elements

Set Equalities using Membership Table

Prove using membership table that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

For very element $x \in U$ there are exactly 8 possibilities based on it's membership (denoted by $0/1$) in some combination of A, B, and C

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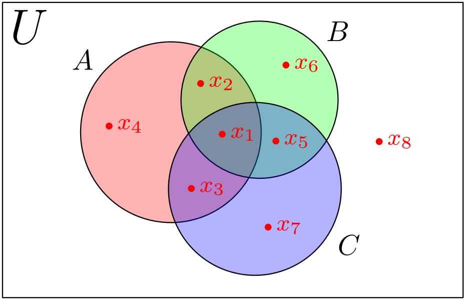

element type	A	B	C
x_1	1	1	1
x_2	1	1	0
x_3	1	0	1
x_4	1	0	0
x_5	0	1	1
x_6	0	1	0
x_7	0	0	1
x_8	0	0	0

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A	B	C					
1	1	1					
1	1	0					
1	0	1					
1	0	0					
0	1	1					
0	1	0					
0	0	1					
0	0	0					

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A	B	C	$B \cup C$				
1	1	1					
1	1	0					
1	0	1					
1	0	0					
0	1	1					
0	1	0					
0	0	1					
0	0	0					

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1	1	1	1				
1	1	0	1				
1	0	1	1				
1	0	0	0				
0	1	1	1				
0	1	0	1				
0	0	1	1				
0	0	0	0				

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A	B	C	$B \cup C$	$A \cap (B \cup C)$			
1	1	1	1				
1	1	0	1				
1	0	1	1				
1	0	0	0				
0	1	1	1				
0	1	0	1				
0	0	1	1				
0	0	0	0				

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1	1	1	1	1			
1	1	0	1	1			
1	0	1	1	1			
1	0	0	0	0			
0	1	1	1	0			
0	1	0	1	0			
0	0	1	1	0			
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1	1	1	1	1			
1	1	0	1	1			
1	0	1	1	1			
1	0	0	0	0			
0	1	1	1	0			
0	1	0	1	0			
0	0	1	1	0			
0	0	0	0	0			

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A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$		
1	1	1	1	1	1		
1	1	0	1	1	1		
1	0	1	1	1	0		
1	0	0	0	0	0		
0	1	1	1	0	0		
0	1	0	1	0	0		
0	0	1	1	0	0		
0	0	0	0	0	0		

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A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	
1	1	1	1	1	1		
1	1	0	1	1	1		
1	0	1	1	1	0		
1	0	0	0	0	0		
0	1	1	1	0	0		
0	1	0	1	0	0		
0	0	1	1	0	0		
0	0	0	0	0	0		

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A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	
1	1	1	1	1	1	1	
1	1	0	1	1	1	0	
1	0	1	1	1	0	1	
1	0	0	0	0	0	0	
0	1	1	1	0	0	0	
0	1	0	1	0	0	0	
0	0	1	1	0	0	0	
0	0	0	0	0	0	0	

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A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
1	1	1	1	1	1	1	
1	1	0	1	1	1	0	
1	0	1	1	1	0	1	
1	0	0	0	0	0	0	
0	1	1	1	0	0	0	
0	1	0	1	0	0	0	
0	0	1	1	0	0	0	
0	0	0	0	0	0	0	

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A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

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A	B	C	$B \cup C$	$A \cap (B \cup C)$	$A \cap B$	$A \cap C$	$(A \cap B) \cup (A \cap C)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

For each type of element in U the entries in red columns are the same

Set Equalities using Membership Table

Prove using membership table that $(A \setminus C) \cup (B \setminus C) = (A \cup B) \setminus C$

Set Equalities using Membership Table

Prove using membership table that $(A \setminus C) \cup (B \setminus C) = (A \cup B) \setminus C$

A	B	C					
1	1	1					
1	1	0					
1	0	1					
1	0	0					
0	1	1					
0	1	0					
0	0	1					
0	0	0					

Set Equalities using Membership Table

Prove using membership table that $(A \setminus C) \cup (B \setminus C) = (A \cup B) \setminus C$

A	B	C	$A \setminus C$				
1	1	1					
1	1	0					
1	0	1					
1	0	0					
0	1	1					
0	1	0					
0	0	1					
0	0	0					

Set Equalities using Membership Table

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A	B	C	$A \setminus C$				
1	1	1	0				
1	1	0	1				
1	0	1	0				
1	0	0	1				
0	1	1	0				
0	1	0	0				
0	0	1	0				
0	0	0	0				

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A	B	C	$A \setminus C$	$B \setminus C$			
1	1	1	0				
1	1	0	1				
1	0	1	0				
1	0	0	1				
0	1	1	0				
0	1	0	0				
0	0	1	0				
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A	B	C	$A \setminus C$	$B \setminus C$			
1	1	1	0	0			
1	1	0	1	1			
1	0	1	0	0			
1	0	0	1	0			
0	1	1	0	0			
0	1	0	0	1			
0	0	1	0	0			
0	0	0	0	0			

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A	B	C	$A \setminus C$	$B \setminus C$	$(A \setminus C) \cup (B \setminus C)$		
1	1	1	0	0			
1	1	0	1	1			
1	0	1	0	0			
1	0	0	1	0			
0	1	1	0	0			
0	1	0	0	1			
0	0	1	0	0			
0	0	0	0	0			

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A	B	C	$A \setminus C$	$B \setminus C$	$(A \setminus C) \cup (B \setminus C)$		
1	1	1	0	0	0		
1	1	0	1	1	1		
1	0	1	0	0	0		
1	0	0	1	0	1		
0	1	1	0	0	0		
0	1	0	0	1	1		
0	0	1	0	0	0		
0	0	0	0	0	0		

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A	B	C	$A \setminus C$	$B \setminus C$	$(A \setminus C) \cup (B \setminus C)$	$A \cup B$	
1	1	1	0	0	0		
1	1	0	1	1	1		
1	0	1	0	0	0		
1	0	0	1	0	1		
0	1	1	0	0	0		
0	1	0	0	1	1		
0	0	1	0	0	0		
0	0	0	0	0	0		

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A	B	C	$A \setminus C$	$B \setminus C$	$(A \setminus C) \cup (B \setminus C)$	$A \cup B$	
1	1	1	0	0	0	1	
1	1	0	1	1	1	1	
1	0	1	0	0	0	1	
1	0	0	1	0	1	1	
0	1	1	0	0	0	1	
0	1	0	0	1	1	1	
0	0	1	0	0	0	0	
0	0	0	0	0	0	0	

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A	B	C	$A \setminus C$	$B \setminus C$	$(A \setminus C) \cup (B \setminus C)$	$A \cup B$	$(A \cup B) \setminus C$
1	1	1	0	0	0	1	
1	1	0	1	1	1	1	
1	0	1	0	0	0	1	
1	0	0	1	0	1	1	
0	1	1	0	0	0	1	
0	1	0	0	1	1	1	
0	0	1	0	0	0	0	
0	0	0	0	0	0	0	

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1	1	1	0	0	0	1	0
1	1	0	1	1	1	1	1
1	0	1	0	0	0	1	0
1	0	0	1	0	1	1	1
0	1	1	0	0	0	1	0
0	1	0	0	1	1	1	1
0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0

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A	B	C	$A \setminus C$	$B \setminus C$	$(A \setminus C) \cup (B \setminus C)$	$A \cup B$	$(A \cup B) \setminus C$
1	1	1	0	0	0	1	0
1	1	0	1	1	1	1	1
1	0	1	0	0	0	1	0
1	0	0	1	0	1	1	1
0	1	1	0	0	0	1	0
0	1	0	0	1	1	1	1
0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0

Set Equalities using Logical Equivalence

Two sets are equal if and only if they have the same elements

$$A = B \quad \text{means} \quad \forall x \, (x \in A \leftrightarrow x \in B)$$

Set Equalities using Logical Equivalence

Two sets are equal if and only if they have the same elements

$$A = B \quad \text{means} \quad \forall x\,(x \in A \leftrightarrow x \in B)$$

- To prove two sets R and S to be equal
- Prove that the membership predicate of R is logically equivalent to the membership predicate of S
- Recall the membership predicate decides whether or not x is in a set
- When the two membership predicates are logically equivalent, for any x they will either both be True or both be False
- We get that $\forall x\,(x \in R \leftrightarrow x \in S)$ is true

Set Equalities using Logical Equivalence

Prove using logical equivalences that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Set Equalities using Logical Equivalence

Prove using logical equivalences that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$x \in A \cap (B \cup C)$ $\triangleright$ LHS

Set Equalities using Logical Equivalence

Prove using logical equivalences that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$x \in A \cap (B \cup C)$ ▷ LHS

$\equiv x \in A \wedge x \in (B \cup C)$ ▷ Intersection

Set Equalities using Logical Equivalence

Prove using logical equivalences that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$x \in A \cap (B \cup C)$ ▷ LHS

$\equiv x \in A \wedge x \in (B \cup C)$ ▷ Intersection

$\equiv x \in A \wedge (x \in B \vee x \in C)$ ▷ Union

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Prove using logical equivalences that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$x \in A \cap (B \cup C)$ ▷ LHS

$\equiv x \in A \wedge x \in (B \cup C)$ ▷ Intersection

$\equiv x \in A \wedge (x \in B \vee x \in C)$ ▷ Union

$\equiv (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)$ ▷ Distributive Law

Set Equalities using Logical Equivalence

Prove using logical equivalences that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$$x \in A \cap (B \cup C) \qquad \triangleright \text{ LHS}$$

$$\equiv x \in A \wedge x \in (B \cup C) \qquad \triangleright \text{ Intersection}$$

$$\equiv x \in A \wedge (x \in B \vee x \in C) \qquad \triangleright \text{ Union}$$

$$\equiv (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C) \qquad \triangleright \text{ Distributive Law}$$

$$\equiv x \in (A \cap B) \vee x \in (A \cap C) \qquad \triangleright \text{ Intersection}$$

Set Equalities using Logical Equivalence

Prove using logical equivalences that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$x \in A \cap (B \cup C)$ ▷ LHS

$\equiv x \in A \wedge x \in (B \cup C)$ ▷ Intersection

$\equiv x \in A \wedge (x \in B \vee x \in C)$ ▷ Union

$\equiv (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)$ ▷ Distributive Law

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Set Equality using Subset Relations

Two sets R and S are equal if $R \subseteq S$ and $S \subseteq R$

Set Equality using Subset Relations

Two sets R and S are equal if $R \subseteq S$ and $S \subseteq R$

- To prove $R = S$
- Prove $R \subseteq S$ and
- Prove $S \subseteq R$
- By the above definition we get that $R = S$

Set Equality using Subset Relations

Proving using subset relations that $\overline{(A \cup C) \cap B} = \overline{B} \cup (\overline{C} \cap \overline{A})$

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First show that

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Next show that

2 $\overline{B} \cup (\overline{C} \cap \overline{A}) \subseteq \overline{(A \cup C) \cap B}$

Set Equality using Subset Relations

We need to prove that

$\boxed{\mathbf{1}}$ $\overline{(A \cup C) \cap B} \subseteq \overline{B} \cup (\overline{C} \cap \overline{A})$ $\boxed{\mathbf{2}}$ $\overline{B} \cup (\overline{C} \cap \overline{A}) \subseteq \overline{(A \cup C) \cap B}$

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ICP-4-26 Prove **1** : if $x \in \overline{(A \cup C) \cap B}$, then $x \in \overline{B} \cup (\overline{C} \cap \overline{A})$

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- $x \notin B$
  - $x \in \overline{B}$
  - $x \in \overline{B} \cup (\overline{C} \cap \overline{A})$
- $x \notin (A \cup C)$
  - $x \notin A \wedge x \notin C$

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- $x \notin B$
  - $x \in \overline{B}$
  - $x \in \overline{B} \cup (\overline{C} \cap \overline{A})$
- $x \notin (A \cup C)$
  - $x \notin A \wedge x \notin C$
  - $x \in \overline{A} \cap \overline{C}$
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Set Equality

- Equality of two sets can be proved using
  - Algebraic Rules (Set Identities)
  - Set Membership Tables
  - Logical Equivalence of membership predicates
  - By proving bidirectional subset relationships