Set Theory

- Sets: Definition, Universal Set, Complement, Cardinality
- Subset and Power Set
- Sets Operations
- Set Equality
- Characteristic Vectors: Sets as Bit-Vectors
- Multisets

Imdad ullah Khan

Set Operations

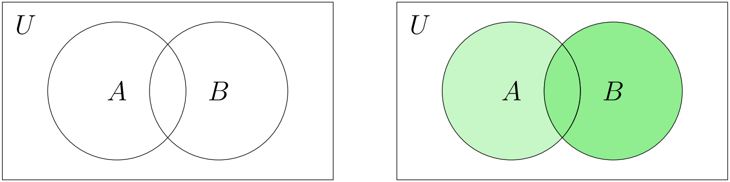
- Set operations (typically) take two sets and return another set
  - Set complement is a unary operation (takes one set)
  - Universal set is also involved in the background
- Set Algebra is built upon these set operations

Set Operations: Union

The union of two sets A and B is the set containing all elements that are in A or B or both

$$A \cup B = \{x | x \in A \vee x \in B\}$$

$$\{1, 3, 5\} \cup \{1, 5, 6, 7\} = \{1, 3, 5, 6, 7\}$$



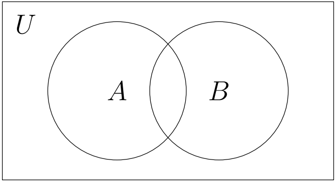
Two sets A and B $\qquad$ $A \cup B$

Set Operations: Intersection

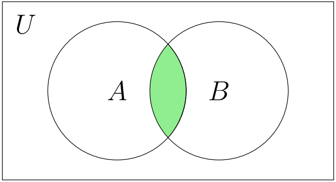
The intersection of two sets A and B is the set containing all elements that are both in A and B

$$A \cap B \; = \; \{x | x \in A \wedge x \in B\}$$

$$\{1, 3, 5\} \cap \{1, 5, 6, 7\} \; = \; \{1, 5\}$$



Two sets A and B



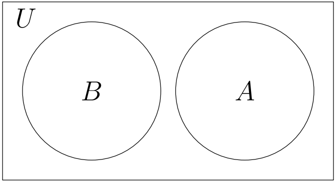
$A \cap B$

Disjoint Sets

Two sets A and B are disjoint if $A \cap B = \emptyset$

No elements in common. Logical expression!

$\{1, 3, 5\} \cap \{2, 4, 6\} = \emptyset$



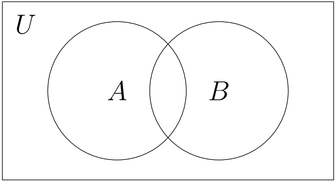
Disjoint sets A and B

Set Operations: Set Difference

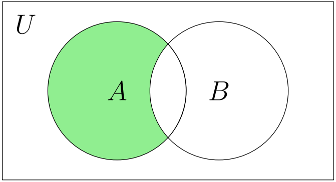
The difference of two sets A and B is the set containing those elements that are in A but not in B

$$A \setminus B \;=\; \{x | x \in A \wedge x \notin B\}$$

$$\{1,3,5\} \setminus \{1,5,6,7\} \;=\; \{3\}$$



Two sets A and B



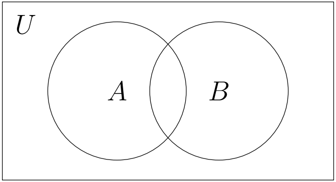
$A \setminus B$

Set Operations: Symmetric Difference

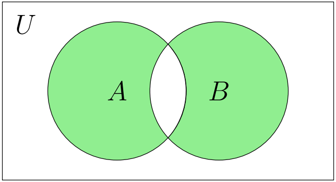
The symmetric difference of two sets A and B is the set containing those elements that are in exactly one of the two sets

$$A \oplus B \ = \ \{x | x \in A \oplus x \in B\}$$

$$\{1, 3, 5\} \oplus \{1, 5, 6, 7\} \ = \ \{3, 6, 7\}$$



Two sets A and B

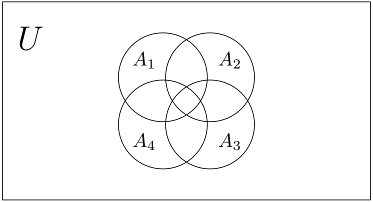


$A \oplus B$

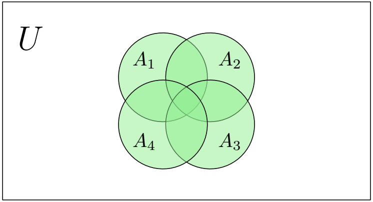
Generalized Union

The union of a collection of sets is the set containing those elements that are members of at least one set in the collection

$$A_1 \cup A_2 \cup \ldots \cup A_n = \bigcup_{i=1}^{n} A_i = \{x \mid \exists\, i\; x \in A_i\}$$



n sets $A_1, A_2, \ldots, A_n$



$A_1 \cup A_2 \cup A_3 \cup A_4$

Generalized Union

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$$A_1 \cup A_2 \cup \ldots \cup A_n = \bigcup_{i=1}^{n} A_i = \{x \mid \exists\, i\; x \in A_i\}$$

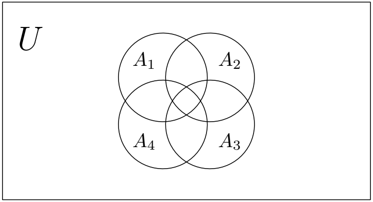
- Let $A_i = \{i, i+1, i+2, \ldots\}$
  - $A_1 = \{1, 2, 3, \ldots\}$
  - $A_2 = \{2, 3, 4, \ldots\}$
  - **ICP 4-17** $A_3 = ?$
  - **ICP 4-18** $A_4 = ?$

ICP 4-19 $\bigcup_{i=1}^{n} A_i = \{1, 2, 3, \ldots\}$

Generalized Intersection

The intersection of a collection of sets is the set containing those elements that are members of all the sets in the collection

$$A_1 \cap A_2 \cap \ldots \cap A_n \; = \; \bigcap_{i=1}^{n} A_i \; = \; \{x | \; \forall \; i \; x \in A_i\}$$



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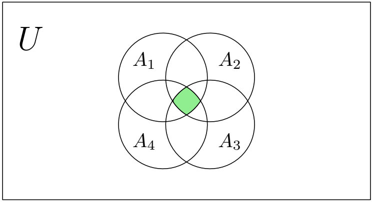

$A_1 \cap A_2 \cap A_3 \cap A_4$

Generalized Intersection

The intersection of a collection of sets is the set containing those elements that are members of all the sets in the collection

$$A_1 \cap A_2 \cap \ldots \cap A_n = \bigcap_{i=1}^{n} A_i = \{x \mid \forall\ i\ x \in A_i\}$$

- Let $A_i = \{i, i+1, i+2, \ldots\}$
  - $A_1 = \{1, 2, 3, \ldots\}$
  - $A_2 = \{2, 3, 4, \ldots\}$
  - **ICP 4-20** $A_5 = ?$
  - **ICP 4-21** $A_6 = ?$

ICP 4-22 $\bigcap_{i=1}^{n} A_i = \{n, n+1, n+2, \ldots\}$

Set Operations

- Set Operation (Binary)
  - Union
  - Intersection
  - Difference
  - Symmetric Difference
- Generalized Union
- Generalized Intersection