

## Predicate Logic

- Predicates and Propositional Functions
- Universal and Existential Quantifiers
- Negating Quantified Statements
- Nested Quantified Expressions

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# Proposition

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A statement is a description of something

A proposition is a statement that is either **true** or **false**

Recall that the following are not propositions

$$x = 4, \quad x > y + 3, \quad x + y = z$$

▷ Their truth values depend on values of the variables

These are actually predicates

## Predicate

A predicate is a property that is true or false about the subject

$x$  is greater than 3

$y$  is equal to  $z$

# Predicate

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## Predicate

A predicate is a property that is true or false about the subject

$x$  is greater than 3  
*subject*      *predicate*

$y$  is equal to  $z$   
*subject*      *predicate*

## Propositional function

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$x$  is greater than 3  
*subject* *predicate*

- Denoted by  $P(x)$
- $P$  denotes the predicate “is greater than 3”
- $x$  is the variable (also called subject or argument)
- $P(x)$  is the value of the propositional function  $P$  at  $x$

# Propositional function

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## A Propositional function

- Takes one or more arguments ▷ multivariable functions
- Stands for the property (predicate) of the variable(s)
- Yields the value **true (T)** or **false (F)** for the subject(s)
- Becomes a proposition when variable(s) are given value(s)

## Propositional function

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$$P(x) : x > 5$$

**ICP 2-1**  $P(3) :$  ▷ **T/F**

**ICP 2-2**  $P(7) :$  ▷ **T/F**

**ICP 2-3**  $P(5) :$  ▷ **T/F**

## Propositional function

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$$Q(x, y) : x > y$$

**ICP 2-4**

$Q(3, 4) :$

▷ **T/F**

**ICP 2-5**

$Q(5, 4) :$

▷ **T/F**

**ICP 2-6**

$Q(4, 4) :$

▷ **T/F**



## Propositional function

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$P(x)$  :  $x$  is a professor at LUMS

**ICP 2-7**

$P(\text{Imdad})$  :

▷ **T/F**

**ICP 2-8**

$P(\text{Pythagoras})$  :

▷ **T/F**

**ICP 2-9**

$P(\text{Jahan})$  :

▷ **T/F**

## Propositional function

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$Q(x, y, z) : x$  teaches course  $y$  in university  $z$

**ICP 2-10**

$Q(\text{Imdad, Calc, ITU}) :$

▷ **T/F**

**ICP 2-11**

$Q(\text{Pythagoras, Bio, PU}) :$

▷ **T/F**

**ICP 2-12**

$Q(\text{Jahan, DM, LUMS}) :$

▷ **T/F**

## Propositional function

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$$Q(x, y) : x > y$$

**ICP 2-13**

$Q(10, 7) :$

▷ **T/F**

**ICP 2-14**

$Q(\text{cat}, 4) :$

▷ **T/F**

**ICP 2-15**

$Q(\text{course}, \text{mountain}) :$

▷ **T/F**

# Universe of Discourse

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We need to have a universe of discourse

**UoD: A collection of subjects; things we are talking about**

With a clear UoD, for each value of the variable predicates become propositions

Each variable may have a different universe of discourse

**ICP 2-16**

What is an appropriate UoD for the variable  $x$  in

$$P(x) : x > 4 ?$$

- a) Set of people
- b) Set of real numbers
- c) Set of cats

**ICP 2-17**

What is an appropriate UoD for the variable  $x$  in

$P(x)$  :  $x$  is a professor at LUMS ?

- a) Set of people
- b) Set of real numbers
- c) Set of cats

**ICP 2-18**

What are appropriate UoDs for the variables  $x$  and  $y$  in,

$$Q(x, y) : x > y ?$$

- a) Set of people and set of real numbers
- b) Set of real numbers and set of cats
- c) Set of real numbers and set of real numbers

### ICP 2-19

What are appropriate UoDs for the variables  $x, y$ , and  $z$  in,  
 $Q(x, y, z) : x$  teaches course  $y$  in university  $z$  ?

- a) People, Courses, Universities
- b) Courses, Universities, People
- c) University, People, Courses



## Predicate: Summary

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- A predicate is a property that is true or false about the subject(s)
- $P(x)$  is the value of propositional function  $P$  at  $x$
- Takes one or more variables
- Becomes a proposition when variable(s) are given value(s)
- Universe of discourse is set of possible values for variables
- Each variable may have a different universe of discourse