Logical Equivalence

- Tautology, Contradiction, Logical Equivalence
- Logical Equivalence using Truth Table
- Logical Equivalence using Laws

Imdad ullah Khan

Logical Equivalence

Two compound propositions R and S are equivalent if $R \leftrightarrow S$ is a tautology

- $R \equiv S$, $R \Leftrightarrow S$, R is equivalent to S
- $R \equiv S$ if whenever R is true, S is true and vice-versa
- They have the same truth values
- The definition of equivalence follows from bicondintional statement

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

- $P \leftrightarrow Q$ is true when $P = Q$

Logical Equivalence using Truth Tables

- Two compound propositions are logically equivalent if their truth values are equal for all possible combinations of truth values of atomic propositions (variables)
- More atomic propositions mean larger truth tables

ICP 1-7 How many rows are there in the truth table of a compound proposition made up of n atomic propositions?

a) n

b) $2n$

c) n^2

d) 2^n

Each new proposition doubles the number of rows of truth table

This method becomes difficult and error prone and soon impossible

Logical Equivalence using Laws

- To prove two compound propositions R and S logically equivalent
- Start with one compound proposition (say R) and replace it with an equivalent compound proposition using "established equivalences"
- These established equivalences are called "Laws"
- Continue doing this until we get the compound proposition S

Logical Equivalence using Laws

Equivalence	Name
$p \wedge T \equiv p$ $p \vee F \equiv p$	Idenitity Laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	Domination Laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent Laws

Logical Equivalence using Laws

Equivalence	Name
$\neg(\neg p) \equiv p$	Double Negation Laws
$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	Negation Laws
$\neg(p \vee q) \equiv \neg p \wedge \neg q$ $\neg(p \wedge q) \equiv \neg p \vee \neg q$	De Morgan's Laws
$p \rightarrow q \equiv \neg p \vee q$	Implication Law

Logical Equivalence using Laws

Equivalence	Name
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative Laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative Laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive Laws

Logical Equivalence using Laws

$((s \wedge q) \vee \neg p) \vee (p \rightarrow p) \vee (\neg q \wedge p)$ is a tautology

$((s \wedge q) \vee \neg p) \vee (p \rightarrow p) \vee (\neg q \wedge p)$	Original
$\equiv ((s \wedge q) \vee \neg p) \vee (\neg p \vee p) \vee (\neg q \wedge p)$	Implication Law
$\equiv ((s \wedge q) \vee \neg p) \vee T \vee (\neg q \wedge p)$	Negation Law
$\equiv T \vee (\neg q \wedge p)$	Dominat. Law
$\equiv T$	Dominat. Law

Logical Equivalence Using Laws

ICP 1-8 Is $(q \wedge p) \vee \neg(q \rightarrow p) \equiv q$?

$(q \wedge p) \vee \neg(q \rightarrow p)$	Original
$\equiv (q \wedge p) \vee \neg(\neg q \vee p)$	Implication Law
$\equiv (q \wedge p) \vee (\neg(\neg q) \wedge \neg p)$	De Morgan's Law
$\equiv (q \wedge p) \vee (q \wedge \neg p)$	Double Negation Law
$\equiv q \wedge (p \vee \neg p)$	Distributive Law
$\equiv q \wedge T$	Negation Law
$\equiv q$	Domination Law

Logical Equivalence Using Laws

ICP 1-9 Is $(P \to R) \vee (Q \to R) \equiv (P \wedge Q) \to R$?

$(P \to R) \vee (Q \to R)$	Original LHS
$\equiv (\neg P \vee R) \vee (Q \to R)$	Implication Law
$\equiv (\neg P \vee R) \vee (\neg Q \vee R)$	Implication Law
$\equiv (\neg P \vee \neg Q) \vee (R \vee R)$	Associative Law
$\equiv (\neg P \vee \neg Q) \vee R$	Idempotent Law
$\equiv \neg(P \wedge Q) \vee R$	DeMorgan's Law
$\equiv (P \wedge Q) \to R$	Implication Law
$=$ RHS	

Logical Equivalence using Laws: Summary

- Logical Equivalence Laws are established equivalences
- Verify using truth tables that they are indeed equivalent
- To show equivalence of compound propositions R and S
- Start with one (say R) and with a series of applications of equivalence laws derive S