Logical Equivalence

- Tautology, Contradiction, Logical Equivalence
- Logical Equivalence using Truth Table
- Logical Equivalence using Laws

Imdad ullah Khan

Logical Equivalence

Two compound propositions R and S are equivalent if $R \leftrightarrow S$ is a tautology

- $R \equiv S$, $R \Leftrightarrow S$, R is equivalent to S
- $R \equiv S$ if whenever R is true, S is true and vice-versa
- They have the same truth values
- The definition of equivalence follows from bicondintional statement

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

- $P \leftrightarrow Q$ is true when $P = Q$

Logical Equivalence using Truth Tables

- To test for logical equivalence of two compound propositions R and S

- Construct a truth table including every variable (atomic propositions)

- Populate the column of $R \leftrightarrow S$ and see if it is a tautology

- The last step basically means to check if the corresponding values of the R and S are equal

Logical Equivalence using Truth Tables

ICP 1-3   Is $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$?

P	Q	$\neg P$	$\neg Q$	$\neg P \vee \neg Q$	$P \wedge Q$	$\neg(P \wedge Q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

Notice the corresponding columns are equal, equivalence follows.

This is called the DeMorgan's Law

Logical Equivalence using Truth Tables

$$P \rightarrow Q \equiv \neg Q \rightarrow \neg P$$

P	Q	$P \rightarrow Q$	$\neg P$	$\neg Q$	$\neg Q \rightarrow \neg P$	$P \rightarrow Q \leftrightarrow \neg Q \rightarrow \neg P$
T	T	T	F	F	T	T
T	F	F	F	T	F	T
F	T	T	T	F	T	T
F	F	T	T	T	T	T

Logical Equivalence using Truth Tables

ICP 1-4   Is $P \rightarrow Q \equiv \neg P \vee Q$?

P	Q	$\neg P$	$P \rightarrow Q$	$\neg P \vee Q$	$P \rightarrow Q \leftrightarrow \neg P \vee Q$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Logical Equivalence using Truth Tables

ICP 1-5   Is $(P \rightarrow R) \vee (Q \rightarrow R) \equiv (P \wedge Q) \rightarrow R$?

P	Q	R	$P \rightarrow R$	$Q \rightarrow R$	$(P \rightarrow R) \vee (Q \rightarrow R)$	$P \wedge Q$	$(P \wedge Q) \rightarrow R$
T	T	T	T	T	T	T	T
T	T	F	F	F	F	T	F
T	F	T	T	T	T	F	T
T	F	F	F	T	T	F	T
F	T	T	T	T	T	F	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

ICP 1-6 Is $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee ((P \wedge R)$?

P	Q	R	$Q \vee R$	$P \wedge$ $(Q \vee R)$	$P \wedge Q$	$P \wedge R$	$(P \wedge Q)$ $\vee (P \wedge R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

This is called the Distributive Law of AND over OR

Logical Equivalence using Laws: Summary

- To show equivalence of compound propositions R and S

- Make truth table of R and S and verify that corresponding entries are equal

- Alternatively, verify that the biimplication between R and S is a tautology