Logical Equivalence

- Tautology, Contradiction, Logical Equivalence
- Logical Equivalence using Truth Table
- Logical Equivalence using Laws

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Compound Proposition: Recap

- Negation of a proposition
- Proposition made by combining two propositions with AND, OR, XOR, IF-THEN, IFF
- Can make compound propositions from others
- Compound proposition $\rightarrow$ Truth Table
- Truth Table $\rightarrow$ Compound Proposition

Today we will discuss equivalence of two compound propositions

Tautology, Contradiction, Contingency

Tautology

A compound proposition whose truth value is **T** for all possible truth values of its atomic propositions

Contradiction

A compound proposition whose truth value is **F** for all possible truth values of its atomic propositions

Contingency

A compound proposition whose truth value is not a constant (neither a tautology nor a contradiction)

Tautology, Contradiction, Contingency

Tautology

P	$\neg P$	$P \vee \neg P$
T	F	**T**
F	T	**T**

Contradiction

P	$\neg P$	$P \wedge \neg P$
T	F	**F**
F	T	**F**

Tautology and contradiction in Computer Programs

while $i \leq 10$ OR $5 * i > 50$ **do**
  $i \leftarrow i + 1$

if $num > i$ **then**
  if $num = i$ **then**
    PRINT("I was here")

How many times the loop iterates?

How many times the PRINT statement is executed?

Tautology, Contradiction, Contingency

IN-Class Problem 1-1 Is $(\neg P \wedge (P \vee Q)) \rightarrow Q$ a tautology?

P	Q	$\neg P$	$P \vee Q$	$\neg P \wedge (P \vee Q)$	$(\neg P \wedge (P \vee Q)) \rightarrow Q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

ICP 1-2 Is $(\neg P \wedge (P \vee Q)) \wedge \neg Q$ a contradiction?

P	Q	$\neg P$	$P \vee Q$	$\neg P \wedge (P \vee Q)$	$\neg Q$	$(\neg P \wedge (P \vee Q)) \wedge \neg Q$
T	T	F	T	F	F	F
T	F	F	T	F	T	F
F	T	T	T	T	F	F
F	F	T	F	F	T	F

Logical Equivalence

Two compound propositions R and S are equivalent if $R \leftrightarrow S$ is a tautology

- $R \equiv S$, $R \Leftrightarrow S$, R is equivalent to S
- $R \equiv S$ if whenever R is true, S is true and vice-versa
- They have the same truth values
- The definition of equivalence follows from bicondintional statement

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

- $P \leftrightarrow Q$ is true when $P = Q$

Applications of Logical Equivalence

Logical equivalence is used

- to simplify statements
- to make computer programs efficient yet correct
- to verify if two programs are equivalent
- for circuit minimization

Logical Equivalence: Circuit Minimization

source: skedsoft.com

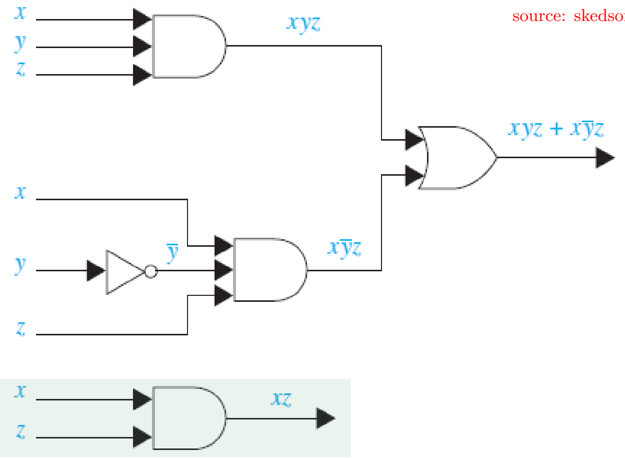
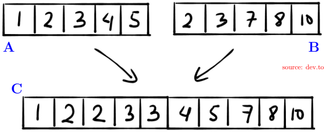


FIGURE 1 **Two Circuits with the Same Output.**

Logical Equivalence: Merging

The following two pieces of code perform exactly the same task

Merge two sorted arrays A and B into a new sorted array C



if $\big[(i+j \leq m+n)$ AND $(i \leq m)$ AND $((j \geq n)$ OR $(A[i] \leq B[j]))\big]$ **then**

$C[k] \leftarrow A[i]$

$i++$

if $\big[((i+j \leq m+n)$ AND $(i \leq m)$ AND $(j \geq n))$ OR $((i+j \leq m+n)$ AND $(i \leq m)$ AND $(A[i] \leq B[j]))\big]$ **then**

$C[k] \leftarrow B[j]$

$j++$

Tautology and Contradiction: Summary

- Defined tautology and contradiction
- Discussed their applications in computer programming
- Define logical equivalence and it's connection to bi-implication
- Saw example of different but equal conditions statements in computer programs