Propositional Logic

- Proposition and truth value
- Compound proposition and truth table
- Implication and it's derivatives

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Compound Propositions: if-then

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

- P implies Q
- P: hypothesis (premise) Q: conclusion (consequence)
- P is sufficient for Q
- Q is necessary for P

Compound Propositions: if-then

If you solve the Goldbach conjecture you will get an A

- P : You solved the Goldbach conjecture
- Q : You get an A
- $P = T$ and $Q = T$
  - $P \rightarrow Q$ should be True
  - The Policy is applied
- $P = T$ and $Q = F$
  - $P \rightarrow Q$ should be False
  - The Policy is violated
- $P = F$ and $Q = T$
  - $P \rightarrow Q$ should be True
  - The Policy is applied
- $P = F$ and $Q = F$
  - $P \rightarrow Q$ should be True ▷ If $1 = 2$, then I am the Pope
  - The Policy is applied

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Converse, Contrapositive, Inverse

We can form other conditional statement from $P \rightarrow Q$

The **converse** of $P \rightarrow Q$ is $Q \rightarrow P$

The **contrapositive** of $P \rightarrow Q$ is $\neg Q \rightarrow \neg P$

The **inverse** of $P \rightarrow Q$ is $\neg P \rightarrow \neg Q$

Converse, Contrapositive, Inverse

I go for a walk only when the weather is sunny

If I go for a walk, then the weather is sunny

P : I go for a walk $\qquad$ Q : The weather is sunny

$P \rightarrow Q$

If I go for a walk, then the weather is sunny $\triangleright$ **Given**

We see whether the converse, contrapositive, and inverse is the same

Converse, Contrapositive, Inverse

P : I go for a walk $\qquad$ Q : The weather is sunny

$P \to Q$

If I go for a walk, then the weather is sunny $\triangleright$ **Given**

$Q \to P$

If the weather is sunny, then I go for a walk $\triangleright$ **No**

$\neg P \to \neg Q$

If I don't go for walk, then weather is not sunny $\triangleright$ **No**

$\neg Q \to \neg P$

If weather is not sunny, then I don't go for walk $\triangleright$ **Same**

Converse, Contrapositive, Inverse

Calculus is a prerequisite for Discrete Math.

To take Discrete Math, you must have taken Calculus

If you take Discrete Math, then you've taken Calculus

P : You take Discrete Math $\qquad$ Q : You've taken Calculus

$P \to Q$

If you take Discrete Math, then you've taken Calculus $\triangleright$ **Given**

We see whether the converse, contrapositive, and inverse is the same

Converse, Contrapositive, Inverse

P : You take D.Math $\qquad$ Q : You've taken Calculus

$P \to Q$

If you take Discrete Math, then you've taken Calculus $\triangleright$ **Given**

$Q \to P$

If you've taken Calculus, then you take Discrete Math $\triangleright$ **No**

$\neg P \to \neg Q$

If you don't take Discrete Math, then you've not taken Calculus $\triangleright$ **No**

$\neg Q \to \neg P$

If you've not taken Calculus, then you don't take Discrete Math $\triangleright$ **Same**

Converse, Contrapositive, Inverse

If x is divisible by 4, then x is even

P : x is divisible by 4 $\qquad\qquad$ Q : x is even

$P \rightarrow Q$

If x is divisible by 4, then x is even $\triangleright$ **Given**

We see whether the converse, contrapositive, and inverse is the same

Converse, Contrapositive, Inverse

P : x is divisible by 4 $\qquad$ Q : x is even

$P \to Q$

If x is divisible by 4, then x is even $\triangleright$ **Given**

$Q \to P$

If x is even, then x is divisible by 4 $\triangleright$ **No**

$\neg P \to \neg Q$

If x is not divisible by 4, then x is not even $\triangleright$ **No**

$\neg Q \to \neg P$

If x is not even, then x is not divisible by 4 $\triangleright$ **Same**

Converse, Contrapositive, Inverse

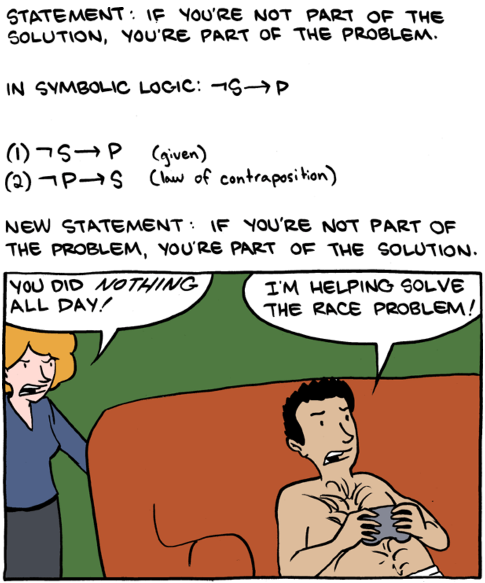
P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg P$	$\neg Q$	$\neg P \to \neg Q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

P	Q	$Q \to P$
T	T	T
T	F	T
F	T	F
F	F	T

P	Q	$\neg P$	$\neg Q$	$\neg Q \to \neg P$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Converse, Contrapositive, Inverse



Implications: Summary

We can form other conditional statement from $P \rightarrow Q$

- $P \rightarrow Q$ is false when P is true and Q is false
- The **converse** of $P \rightarrow Q$ is $Q \rightarrow P$
- The **contrapositive** of $P \rightarrow Q$ is $\neg Q \rightarrow \neg P$
- The **inverse** of $P \rightarrow Q$ is $\neg P \rightarrow \neg Q$
- **An implication is equivalent to it's contrapositive**