Propositional Logic

- Proposition and truth value
- Compound proposition and truth table
- Implication and it's derivatives

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Combining Propositions

A statement is a description of something

A proposition is a statement that is either **true** or **false**

- Recall that for us there is no semantic meaning of a proposition
- For us they are just variables taking the value true or false
- Sometimes called Boolean variables
- We denote them by P, Q, S, etc.

Combining Propositions

A statement is a description of something

A proposition is a statement that is either **true** or **false**

- Clearly, very little can be expressed by propositions only
- Just as in English we can modify, combine and relate statements with words such as "not", "and", "or", "if-then" etc.
- We discuss how to combine propositions
- Except that we will give these connectives precise meanings

Combining Propositions

Problems with English connectives:

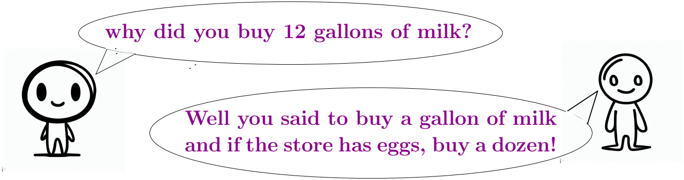
- You may register for CS-210 or CS-212.

  How about both?

- Every student gets a grade.

  Does everyone get the same grade?

  Does everyone get a unique grade?



Compound Propositions: Negation

Let P be a proposition, the truth value of the proposition $\neg P$ is as defined in the following truth table

P	$\neg P$
T	F
F	T

- ("NOT P"), $!P$ ($C++$, Java), $\bar{P}$
- When P is true $\neg P$ is false and vice-versa

Compound Propositions: Negation

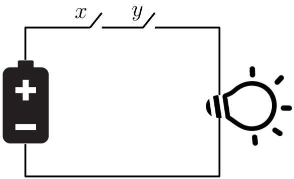
- P : " Today is Friday"
- $\neg P$:
  - "Today is not Friday"
  - "It is not the case that today is Friday"
  - "It is not Friday today"

- $Q : 2 + 2 = 4$
- $\neg Q$:
  - $2 + 2 \neq 4$

Compound Propositions: AND

Let P and Q be propositions, the truth value of the proposition $P \wedge Q$ is defined as follows:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F



- P AND Q, $P \&\& Q$ ($C++$, Java)
- $P \wedge Q$ is true when both P and Q are true

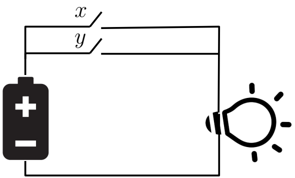
Compound Propositions: AND

- P : " Today is Friday" Q : " It is warm today"
- $P \wedge Q$:
  - "Today is Friday and it is warm"
- $P : 2 + 2 = 4$ $Q : 5 > 1$
- $P \wedge Q$:
  - $(2 + 2 = 4) \wedge (5 > 1)$

Compound Propositions: OR

Let P and Q be propositions, the truth value of the proposition $P \vee Q$ is defined as follows:

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F



- P OR Q, $P||Q$ ($C++$, Java)
- $P \vee Q$ true when one or both of P and Q are true

Compound Propositions: OR

- P : "You may register for CS-210"
  Q : " You may register for CS-212"
- $P \vee Q$:
  - "You may register for CS-210 or CS-212"

- $P : 2 + 2 = 4 \qquad Q : 5 > 1$
- $P \vee Q$:
  - $(2 + 2 = 4) \vee (5 > 1)$

Compound Propositions: XOR

Let P and Q be propositions, the truth value of the proposition $P \oplus Q$ is defined as follows:

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

- P XOR Q, Exclusive OR
- $P \oplus Q$ true when exactly one of P and Q is true

Compound Propositions: XOR

- P : " You may register for CS-210"
  Q : " You may register for CS-212"
- $P \oplus Q$:
  - "You may register for CS-210 or CS-212 but not both"
- $P : 2+2=4 \qquad Q : 5>1$
- $P \oplus Q$:
  - $(2+2=4) \oplus (5>1)$

Compound Propositions: if-then

Let P and Q be propositions, the truth value of the proposition $P \to Q$ is defined as follows:

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

- P implies Q, Conditional Statement if P then Q
- $P \to Q$ is false when P is true and Q is false

Compound Propositions: if-then

- P : " You solve the Goldbach conjecture"
  Q : " You get A in course"
- $P \rightarrow Q$:
  - "If you solve the Goldbach conjecture, then you will get an A in course"

- P : "x is divisible by 4" Q : "x is even"
- $P \rightarrow Q$:
  - if x is divisible by 4, then x is even

Compound Propositions: iff

Let P and Q be propositions, the truth value of the proposition $P \leftrightarrow Q$ is defined as follows:

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

- P iff Q, biconditional Statement
- $P \leftrightarrow Q$ is true when $P = Q$

Compound Propositions: iff

- P : " You solve the Goldbach conjecture"
  Q : " You get an A in course"
- $P \leftrightarrow Q$:
  - "You will get an A in this course iff you solve the Goldbach conjecture"

- P : "x is divisible by 2" Q : "x is even"
- $P \leftrightarrow Q$:
  - x is divisible by 2 iff x is even

Compound Proposition $\rightarrow$ Truth Table

Given a compound proposition, make it's truth table

It gives possible values based on truth values of atomic propositions

Make column for each atomic proposition and compound them to get given proposition

$$\neg Q \vee (\neg P \wedge Q)$$

P	Q	$\neg P$	$\neg Q$	$\neg P \wedge Q$	$\neg Q \vee (\neg P \wedge Q)$
T	T	F	F	F	F
T	F	F	T	F	T
F	T	T	F	T	T
F	F	T	T	F	T

Truth Table $\to$ Compound Proposition

Given a truth table, find a compound proposition for it

P	Q	$P \odot Q$
T	T	F
T	F	T
F	T	T
F	F	F

The true rows method: Our formula should be true, when the input is **exactly one of** the true rows

- Formula is true when P and $\neg Q$ are true OR when $\neg P$ and Q are true
- $(P \wedge \neg Q) \quad \vee \quad (\neg P \wedge Q)$

Truth Table $\rightarrow$ Compound Proposition

Given a truth table, find a compound proposition for it

P	Q	$P \odot Q$
T	T	F
T	F	T
F	T	T
F	F	F

The true rows method: Our formula should be true, when the input is **not any of** the false rows

- Formula is true when NOT (P and Q are true) AND when NOT $\neg P$ and $\neg Q$ are true
- $\neg(P \wedge Q) \quad \wedge \quad \neg(\neg P \wedge \neg Q)$

Truth Table $\rightarrow$ Compound Proposition

Find a logical formula for the following Truth table

P	Q	$P \odot Q$
T	T	F
T	F	T
F	T	T
F	F	F

- This is the truth table of $P \oplus Q$
- The above method will express it in terms of $\wedge, \vee$, or $\neg$

Truth Table → Compound Proposition

Find a logical formula for the truth table of $(P \to Q)$

Find a logical formula for the truth table of $\neg(P \to Q)$

Compound Proposition: Summary

- Negation a proposition
- Proposition made by combining two propositions with AND, OR, XOR, IF-THEN, IFF
- Can make compound propositions from others
- Compound proposition $\rightarrow$ Truth Table
- Truth Table $\rightarrow$ Compound Proposition