

Propositional Logic

- Proposition and truth value
- Compound proposition and truth table
- Implication and it's derivative

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- **Computer Science = Problem Solving**
- Mathematics is at the heart of problem solving
 - definition, modeling, solution, analysis
- Logic is crucial for mathematical reasoning

Logic:

- Fundamental to all mathematical disciplines
- Useful for hardware design, computer architecture
- Artificial Intelligence, Natural Language Processing, Knowledge Representation

Proposition

A statement is a description of something

■ I am an instructor for Discrete Math.

TRUE

■ I am a student in Discrete Math.

FALSE

■ I am lying to you

TRUE / FALSE ?

Proposition

$S = \text{“I am lying to you”}$

- Suppose S is true

Then I am lying,
so S must be false

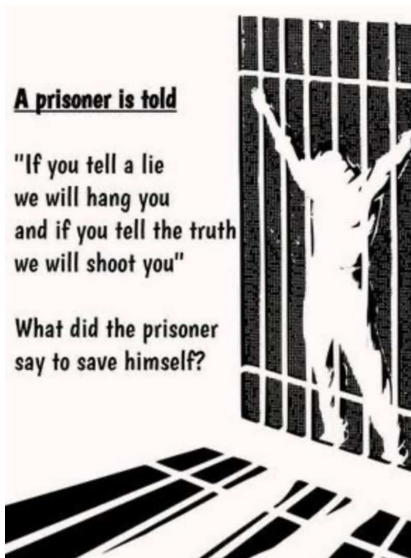
- Suppose S is false

Then I am telling the truth,
so S must be true

I always tell the truth. Even when I lie.

excerpt from Scarface

We will avoid such statements



Proposition

A statement is a description of something

A proposition is a statement that is either **true** or **false**

Does it help?

Proposition

Please give me a higher grade!

■ Statement?

No

■ Proposition?

No

■ Truth Value?

None

Can I get a higher grade?

■ Statement?

No

■ Proposition?

No

■ Truth Value?

None

Proposition

$$1 + 1 = 2$$

■ Statement?

Yes

■ Proposition?

Yes

■ Truth Value?

TRUE

Proposition

$$2 + 2 = 3$$

■ Statement?

Yes

■ Proposition?

Yes

■ Truth Value?

False

Proposition

$$x = 4$$

■ Statement?

Yes

■ Proposition?

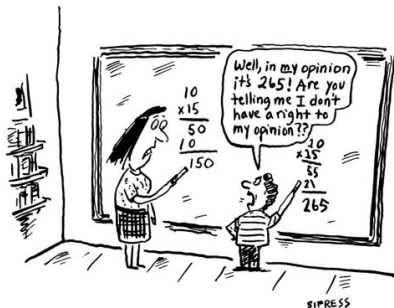
No

■ Truth Value?

None

Proposition

In general, opinions, interrogative and imperative sentences are not propositions



We restrict ourselves to statements that are either true or false

▷ **and not both and not neither**

The truth value of statements

Truth value of statements are not necessarily (easily) known

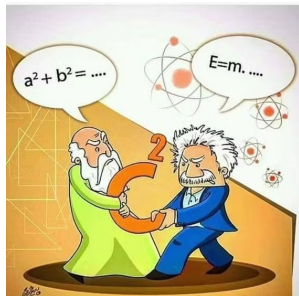
Pythagoras's Theorem (~ 500 BC)

$a^2 + b^2 = c^2$ has solutions where a , b , and c are positive integers

This statement is **TRUE**,

e.g. $a = 3$, $b = 4$, and $c = 5$

Note: strictly speaking this is not proposition but serves our purpose



Credit: ClassHook

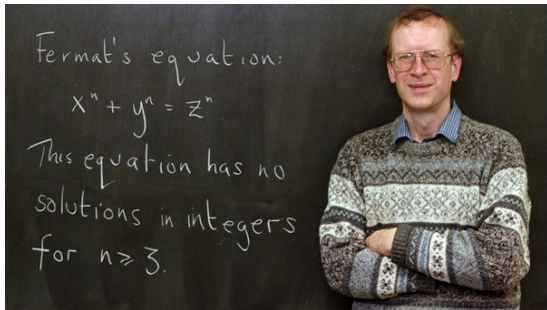
The truth value of statements

Truth value of statements are not necessarily (easily) known

Fermat's Last Theorem (1637)

$a^3 + b^3 = c^3$ has no solution where a, b, c are positive integers

Andrew Wiles (1994) proved this statement to be **TRUE**



The truth value of statements

Truth value of statements are not necessarily (easily) known

Euler Conjecture (1769)

$a^4 + b^4 + c^4 = d^4$ has no solutions where a, b, c, d are positive integers

Noam Elkies (1987) proved this statement **FALSE**

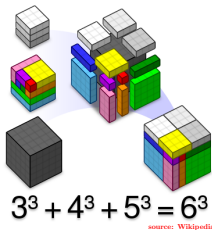
$$a = 2682440,$$

$$b = 15365639,$$

$$c = 18796760,$$

$$d = 20615673,$$

is a solution



Proposition: Summary

- A statement is a description of something
- A proposition is a statement that is either **true** or **false**
- Generally, opinions, interrogative and imperative sentences are not propositions
- Truth values of statements (and propositions) are sometime hard to determine