CS-210 Discrete Mathematics

1. Prove the following DeMorgan's law for sets using membership table.

$$\overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$$

2. Let A, B and C be any sets. Prove the following identity by showing that each side is a subset of the other side.

$$(A \setminus C) \cup (B \setminus C) = (A \cup B) \setminus C.$$

3. Let A, B and C be any sets. Show using set identities that

$$\overline{(A \cup C) \cap B} = \overline{B} \cup (\overline{C} \cap \overline{A}).$$

4. For two sets X and Y, let $A = \mathcal{P}(\overline{X} \cap \overline{Y})$ and $B = \mathcal{P}(\overline{X}) \cap \mathcal{P}(\overline{Y})$.

   Show that $A = B$.

5. Let X, Y and Z be three sets such that Y and Z have the same cardinalities. Define $A = \mathcal{P}(X \times Y)$ and $B = \mathcal{P}(X) \times \mathcal{P}(Z)$.

   Compare the cardinalities of A and B, in what cases are they equal or not.

6. Write down all elements of the following two sets, A and B, where

$$A = \mathcal{P}(\mathcal{P}(\emptyset)),$$

$$B = \mathcal{P}(A),$$

   and

$$C = \mathcal{P}(\mathcal{B}).$$

7. For A, B, $C \subseteq \mathcal{U}$ Prove that

$$A \times (B - C) = (A \times B) - (A \times C)$$

8. Let A, B,C,D be non empty sets.

   (a) Prove that $A \times B \subseteq C \times D$ if and only if $A \subseteq C$ and $B \subseteq D$.

   (b) What happens to the results of (a) if any of the sets A, B,C,D is empty?

9. Let A, B and C be any sets. Prove the following identity using membership table.

$$\overline{(A \cap B) \cup (\overline{A} \cap C)} = (A \cap \overline{B}) \cup (\overline{A} \cap \overline{C})$$

10. Let A and B be any sets. Show using set identities that:

   (a) $\overline{(A \cap \overline{B})} \cup B = \overline{A} \cup B$

   (b) $A \cup (B \setminus A) = A \cup B$

11. Let A, B and C be any sets. Show using set identities that

$$(B \setminus A) \cup (C \setminus A) = (B \cup C) \setminus A$$

12. Let A, B and C be any sets. Show using set identities that

$$\overline{\overline{(A \cup B) \cap C} \cup \overline{B}} = B \cap C$$

13. Prove or disprove each of the following for set A, B $\subseteq \mathcal{U}$

   (a) $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$

   (b) $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$

14. For two sets X and Y, let $A = \mathcal{P}(\overline{X} \cup \overline{Y})$ and $B = \mathcal{P}(\overline{X}) \cup \mathcal{P}(\overline{Y})$.
    Show that $A \supseteq B$.

15. Suppose A, B, and C are sets, and $C \neq \emptyset$;. Prove that if $A \times C = B \times C$, then $A = B$.