**CS-210 Discrete Mathematics**

# Problem Set 8

1. Let $R$ be a relation on the set $A$. Consider the statement

"If $R$ is symmetric and transitive, then $R$ is reflexive"

Find the error in the following claimed proof of the above statement.

**Proof:** Let $a \in A$. Consider an element $b$, such that $(a, b) \in R$. Since $R$ is symmetric, if $(a, b) \in R$, then $(b, a) \in R$. So we get that both $(a, b) \in R$ and $(b, a) \in R$. Now because $R$ is transitive we get that $(a, a) \in R$. Since this is true for any element $a \in A$, we get that $(a, a) \in R$ for every element $a \in A$. Hence $R$ is reflexive.

2. Let $R$ be a relation on $\mathbb{Z}^+ \times \mathbb{Z}^+$, such that $((a, b), (c, d)) \in R \leftrightarrow a + d = b + c$, i.e. $R = \{((a, b), (c, d)) : a + d = b + c\}$. Note that $R$ is a relation on $\mathbb{Z}^+ \times \mathbb{Z}^+$ so it will be a subset of $(\mathbb{Z}^+ \times \mathbb{Z}^+) \times (\mathbb{Z}^+ \times \mathbb{Z}^+)$. Show that $R$ is an equivalence relation.

3. Let $R$ be a relation on $\mathbb{Z} \times \mathbb{Z}$ such that $((a, b), (c, d)) \in R \leftrightarrow ad = bc$, i.e. $R = \{((a, b), (c, d)) : ad = bc\}$.

   - Prove or disprove that $R$ is an equivalence relation.
     Let $\mathbb{Y} = \mathbb{Z} \setminus \{0\}$ (all integers except 0).

     If $R$ is the same relation as above but on the set $\mathbb{Z} \times \mathbb{Y}$, then
   - Prove or disprove that $R$ is an equivalence relation.

4. Let $R$ be a symmetric relation. Show that $R^n$ is a symmetric for all positive integers $n$.

5. Show that if $R$ and $S$ are both $n$-ary relations, then $P_{i_1,i_2,\ldots,i_m}(R \cup S) = P_{i_1,i_2,\ldots,i_m}(R) \cup P_{i_1,i_2,\ldots,i_m}(S)$

6. Construct the table obtained by applying applying the join operator $J_2$ to the relations in Tables 1 and 2.

**TABLE 1 Part_needs**

| Supplier | Part_number | Project |
|---|---|---|
| 23 | 1092 | 1 |
| 23 | 1101 | 3 |
| 23 | 9048 | 4 |
| 31 | 4975 | 3 |
| 31 | 3477 | 2 |
| 32 | 6984 | 4 |
| 32 | 9191 | 2 |
| 33 | 1001 | 1 |

**TABLE 2 Parts_inventory**

| Part_number | Project | Quantity | Color_code |
|---|---|---|---|
| 1001 | 1 | 14 | 8 |
| 1092 | 1 | 2 | 2 |
| 1101 | 3 | 1 | 1 |
| 3477 | 2 | 25 | 2 |
| 4975 | 3 | 6 | 2 |
| 6984 | 4 | 10 | 1 |
| 9048 | 4 | 12 | 2 |
| 9191 | 2 | 80 | 4 |

7. Recall the definition of congruence mod $m$, i.e.

$$a \equiv b \pmod{m} \text{ if and only if } m|(a - b)$$

or equivalently

$$a \equiv b \pmod{m} \text{ if and only if } (a \bmod m) = (b \bmod m).$$

Let $m \geq 4$ be an integer. Let $\mathbb{Z}_m$ be a binary relation on the set of integers defined as follows:

$$\mathbb{Z}_m = \{(x, y) : x \equiv y \pmod{m}\}.$$

- Show that $\mathbb{Z}_m$ is an equivalence relation for all $m \geq 4$.
- How many equivalence classes of $\mathbb{Z}_m$ are there?
- What are the equivalence classes of $\mathbb{Z}_m$?